ANSWER KEY & MARKING SCHEME · CBSE CLASS 10

Real Numbers

Mathematics · Chapter 1 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Multiple Choice (1 mark each, 5 Qs)

Q1. The HCF of 96 and 404 is ____ . [1 mark]

Ans: 4

Q2. Decimal expansion of 13/3125 is ____ . [1 mark]

Ans: Terminating (= 0.00416)

Q3. Which of these is irrational? (a) $\sqrt{16}$ (b) $\sqrt{20}$ (c) 22/7 (d) 0.252525⋯ [1 mark]

Ans: (b) $\sqrt{20} = 2\sqrt{5}$; $\sqrt{5}$ irrational ⇒ product irrational

Q4. If HCF(a, b) = 1, then LCM(a, b) = ____ . [1 mark]

Ans: a · b (co-prime ⇒ LCM = product)

Q5. Prime factorisation of 156 = ____ . [1 mark]

Ans: $2^2 \cdot 3 \cdot 13$

Section B — Very Short Answer (2 marks each, 4 Qs)

Q6. Find HCF and LCM of 12 and 18 by prime factorisation, and verify HCF × LCM = 12 × 18. [2 marks]

Ans: $12 = 2^2 \cdot 3$; $18 = 2 \cdot 3^2$. HCF = 2·3 = 6 ; LCM = $2^2 \cdot 3^2$ = 36. Check: 6 × 36 = 216 = 12 × 18 ✓

Q7. Find the largest number that divides 70 and 125, leaving remainders 5 and 8 respectively. [2 marks]

Ans: Subtract remainders: 65 and 117. HCF(65, 117) = 13. Therefore largest number = 13.

Q8. Show that the number 5n + 7 (n a natural number) cannot end with the digit 0. [2 marks]

Ans: For a number to end in 0, both 2 and 5 must divide it. 5n + 7 has 5n divisible by 5, plus 7. 7 is not divisible by 5, so the sum is not divisible by 5. Hence it cannot end in 0.

Q9. State Theorem 1.3 and explain its role in irrationality proofs. [2 marks]

Ans: Theorem 1.3: If a prime p divides a^2, then p divides a. Role: in proofs like 'prove $\sqrt{p}$ irrational', we get p divides a^2 then invoke Theorem 1.3 to conclude p divides a. Used twice in each such proof (for a, then for b). CBSE marking scheme awards a separate mark for citing it.

Section C — Short Answer (3 marks each, 3 Qs)

Q10. Prove that $\sqrt{5}$ is irrational. [3 marks]

Ans: Assume $\sqrt{5} = a/b$ in lowest form (co-prime a, b). Then $5b^2 = a^2$. So $5 \mid a^2$ ⇒ by Theorem 1.3, $5 \mid a$. Let a = 5m. Then $5b^2 = 25m^2$ ⇒ $b^2 = 5m^2$ ⇒ $5 \mid b^2$ ⇒ $5 \mid b$. So 5 is a common factor of a, b — contradicts co-prime. ∴ $\sqrt{5}$ is irrational. ∎

Q11. Three bells ring at intervals of 6, 12 and 18 minutes. If they ring together at 6 a.m., when will they next ring together? [3 marks]

Ans: LCM(6, 12, 18) = $2^2 \cdot 3^2$ = 36 minutes. They ring together every 36 minutes ⇒ next at 6:36 a.m.

Q12. Find HCF(510, 92) by prime factorisation. Hence find LCM(510, 92). Verify HCF × LCM = 510 × 92. [3 marks]

Ans: $510 = 2 \cdot 3 \cdot 5 \cdot 17$; $92 = 2^2 \cdot 23$. HCF = 2 ; LCM = $2^2 \cdot 3 \cdot 5 \cdot 17 \cdot 23$ = 23460. Check: 2 × 23460 = 46920 = 510 × 92 ✓

Section D — Long Answer (Case Study, 4 marks each, 2 Qs)

Q13. State the Fundamental Theorem of Arithmetic. Using prime factorisation, find HCF and LCM of 378, 180 and 420. Is HCF × LCM = product of three numbers? Justify. **[4 marks]**

> **Ans:** FTA: every composite is uniquely expressible as product of primes apart from order. $378 = 2 \cdot 3^3 \cdot 7$; $180 = 2^2 \cdot 3^2 \cdot 5$; $420 = 2^2 \cdot 3 \cdot 5 \cdot 7$. HCF (smallest common powers): $2 \cdot 3 = 6$. LCM (greatest of all powers): $2^2 \cdot 3^3 \cdot 5 \cdot 7 = 3780$. Product $= 378 \cdot 180 \cdot 420 = 28576800$; HCF·LCM $= 6 \cdot 3780 = 22680$. NOT equal! The identity HCF × LCM = product holds ONLY for two numbers, not three or more.

Q14. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march? **[4 marks]**

> **Ans:** Required = HCF(616, 32). By prime factorisation: $616 = 2^3 \cdot 7 \cdot 11$; $32 = 2^5$. HCF $= 2^3 = 8$. ALTERNATIVE (Euclid, legacy): $616 = 32 \cdot 19 + 8$; $32 = 8 \cdot 4 + 0 \Rightarrow$ HCF = 8. Maximum number of columns = 8.

★ TOPPER ANSWER TEMPLATES

3 TEMPLATES · MEMORISE THE FORMAT

★ TOPPER TEMPLATE — 3-mark question: 'Prove that √p is irrational' (where p is a prime — typically 2, 3, 5, 7)

Appears in nearly every CBSE board set since 2010

Step 1 [1 mark]	**Assumption & setup**	Let us assume, to the contrary, that $\sqrt{p}$ is rational. Then there exist co-prime integers a and b ($b \neq 0$) such that $\sqrt{p} = a/b$. Squaring: $a^2 = p \cdot b^2$ ... (i)
Step 2 [1 mark]	**Apply Theorem 1.3 + substitute**	From (i), p divides a^2. By Theorem 1.3 (if a prime p divides a^2, then p divides a), p divides a. So let $a = p \cdot c$ for some integer c. Substituting in (i): $(p \cdot c)^2 = p \cdot b^2 \Rightarrow p^2 \cdot c^2 = p \cdot b^2 \Rightarrow b^2 = p \cdot c^2$. Thus p divides b^2. Again by Theorem 1.3, p divides b.
Step 3 [1 mark]	**Contradiction + conclusion**	Therefore p is a common factor of a and b. But this contradicts our assumption that a and b are co-prime. Hence our supposition is wrong. Therefore $\sqrt{p}$ is irrational. ∎

COMMON LOSS OF MARKS:
- Forgetting 'co-prime' in the setup → loses 0.5 mark.
- Skipping the explicit citation of Theorem 1.3 → loses 0.5–1 mark.
- Not writing the closing line 'therefore √p is irrational' → loses 0.5 mark.

★ TOPPER TEMPLATE — 3-mark question: 'Prove that a + b·√p is irrational, given that √p is irrational' (chained variant)

Common variant — 2010, 2018, 2023

Step 1 [1 mark]	**Assume the opposite**	Suppose, to the contrary, that $a + b \cdot \sqrt{p}$ is rational. Call this rational number r. Then $a + b \cdot \sqrt{p} = r$.
Step 2 [1 mark]	**Isolate √p**	Rearrange: $b \cdot \sqrt{p} = r - a$, so $\sqrt{p} = (r - a) / b$. The right-hand side is a difference of rationals divided by a rational ($b \neq 0$), hence rational.
Step 3 [1 mark]	**Contradiction + conclusion**	But we are GIVEN that $\sqrt{p}$ is irrational. This is a contradiction. So our assumption is wrong. Therefore $a + b \cdot \sqrt{p}$ is irrational. ∎

COMMON LOSS OF MARKS:
- Treating 'irrational' as a tag without using its definition (must say 'cannot be expressed as ratio of integers').
- Forgetting to invoke the GIVEN ('√p is irrational') in the contradiction.

★ TOPPER TEMPLATE — 3-mark word problem: bells ringing at intervals / cyclists meeting again / tankers coinciding etc.

Annual — 2015, 2017, 2019, 2022

Step 1 [1 mark]	**Identify operation**	The events recur at intervals of a, b, c units. The next time they coincide will be at a time that is a common multiple of a, b, c. The SMALLEST such time = LCM(a, b, c).
Step 2 [1 mark]	**Prime factorisation + LCM**	Prime factorise: a = ..., b = ..., c = ... LCM = product of greatest powers of all primes appearing = ... units.
Step 3 [1 mark]	**Express the answer in the question's units + final time**	Therefore the events next coincide after LCM = ... minutes (or hours). If started at, say, 6 a.m., next coincidence is at ... + start time.

COMMON LOSS OF MARKS:
- Using HCF instead of LCM (interpretation error) — loses all 3 marks.
- Forgetting to convert LCM to the time format the question asks for.

MARKING SCHEME — GENERAL NOTES

- Step marks for proof questions: setup (1), apply Theorem 1.3 + algebra (1), contradiction + conclusion (1).
- Word problems: identifying LCM/HCF (1), computation (1), final expression in question's units (1).
- MCQ: full mark for correct option; 0 for wrong. No negative marking in board paper.
- Half-mark deductions common for: missing 'co-prime' in irrationality setup, missing citation of Theorem 1.3, missing final 'therefore' line, missing unit in word-problem answer.