Ready For Boards
10th & 12th Exam Prep

CHAPTER 1

Real Numbers

CBSE Class 10 · Mathematics · Chapter 1

CBSE · Mathematics · Class 10

WHAT THIS CHAPTER DOES

A Master Fundamental Theorem of Arithmetic (FTA).

B Prove √2, √3, √5 are irrational using a 3-step template.

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's mission

1. Master Fundamental Theorem of Arithmetic (FTA).
2. Prove $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational using a 3-step template.
3. Classify decimal expansions (terminating vs repeating).
4. Hit 6/6 marks in board paper for this chapter.

WHY THIS MATTERS

Why this chapter matters

1. Every later chapter (polynomials, algebra, surds) sits on Real Numbers.
2. Highest score-to-effort ratio in the paper — 6 marks for ~3 ideas.
3. Lays the conceptual base for Class 11 sets, sequences, limits.

TOPIC

A

Fundamental Theorem of Arithmetic

Every composite number can be uniquely factorised into primes, apart from the order of the factors.

TOPIC

Four key skills you must master to use the Fundamental Theorem

PRIME FACTORISATION BY TREE

The factor tree is the canonical visual technique for prime-factorising a number. Start with the given number; divide by the smallest prime that divides it (2 first, then 3, then 5, then 7, then 11...); write the quotient and divisor as branches; repeat until you reach 1. Example: 156 → 2 × 78 → 2 × 2 × 39 → 2 × 2 ×

HCF BY SMALLEST COMMON POWERS

Once both numbers are prime-factorised, HCF = product of the SMALLEST powers of each COMMON prime. Example: $72 = 2^3 \times 3^2$ and $120 = 2^3 \times 3 \times 5$. Common primes: 2 (smallest power = 2^3) and 3 (smallest power = 3^1). HCF = $2^3 \times 3 = 24$. CBSE 2-mark questions test this rule directly; reversing 'smallest'

LCM BY GREATEST OF ALL POWERS

LCM = product of the GREATEST powers of ALL primes that appear in either factorisation (not just common ones). Same example: $72 = 2^3 \times 3^2$ and $120 = 2^3 \times 3 \times 5$. All primes appearing: 2 (greatest power = 2^3), 3 (greatest = 3^2), 5 (greatest = 5^1). LCM = $2^3 \times 3^2 \times 5 = 360$. Verify with HCF × LCM = product:

THE HCF × LCM = PRODUCT IDENTITY

For ANY two positive integers a and b: HCF(a,b) × LCM(a,b) = a × b. This is one of the most-tested 2-mark identities in CBSE Class 10. Used to find one quantity from the other three. Example: if HCF(306,657) = 9, then LCM = (306 × 657) / 9 = 201,042 / 9 = 22,338. CRITICAL CAVEAT: the

WORKED EXAMPLE

FTA in action — HCF & LCM of 96 and 404

1. $96 = 2^5 \times 3$
2. $404 = 2^2 \times 101$. Common prime: 2 (smallest power 2^2)
3. HCF = 4. All primes appearing: 2^5, 3, 101
4. LCM = $2^5 \times 3 \times 101 = 9696$.

VERIFY

HCF × LCM = 4 × 9696 = 38784 = 96 × 404 ✓

96

2×48

$2^2 \times 24$

$2^3 \times 12$

$2^4 \times 6$

$2^5 \times 3$

404

2×202

$2^2 \times 101$

TOPIC

B

Useful identity

Holds for any TWO positive integers (NOT for three or more!).

TOPIC

C

Irrational Numbers

A number that CANNOT be written as p/q where p, q are integers and $q \neq 0$.

TOPIC

Four rules governing operations with irrational numbers

SUM/DIFFERENCE OF RATIONAL + IRRATIONAL

Adding a non-zero rational to an irrational number ALWAYS produces an irrational result. Example: 3 + √2 is irrational. Proof by contradiction: if 3 + √2 = r (rational), then √2 = r − 3 = (rational − rational) = rational, contradicting the fact that √2 is irrational. The 'non-zero' qualifier is essential — multiplying by

PRODUCT OF NON-ZERO RATIONAL × IRRATIONAL

Multiplying a NON-ZERO rational by an irrational ALWAYS produces an irrational result. Example: 5√2 is irrational; (3/4) × √7 is irrational. Same contradiction logic: if 5√2 = r (rational), then √2 = r/5 = rational, which contradicts √2 being irrational. The 'non-zero' qualifier is critical: 0 × √2 = 0, which is

SUM/PRODUCT OF TWO IRRATIONALS

Sum or product of TWO irrationals may be rational OR irrational — no general rule. Examples: (1+√2) + (1−√2) = 2 (rational!). √2 × √2 = 2 (rational). But √2 + √3 is irrational (no nice cancellation). √2 × √3 = √6 (irrational). Each case must be checked separately. CBSE 3-mark questions exploit this 'no general rule'

WHY √P IS IRRATIONAL FOR ANY PRIME P

For every prime p (2, 3, 5, 7, 11, 13, ...), √p is irrational. The proof follows the same three-step structure (assume √p = a/b in lowest form → contradiction via Theorem 1.3). This means there are INFINITELY many irrationals constructible by taking square roots of primes. Combined with their arithmetic rules above, this

THEOREM · LOAD-BEARING RESULT

Theorem 1.3 (load-bearing for irrationality proofs)

> *If a prime p divides a^2, then p divides a.*

STATEMENT

If a prime p divides a^2, then p divides a.

WHY THIS MATTERS

- This is THE result that powers every irrationality proof.
- CBSE marking scheme awards a separate mark for citing this explicitly.
- Don't skip it during the proof — citing it costs nothing and saves a mark.

PROOF · MARK-BY-MARK TEMPLATE

Proof — √2 is irrational (3-mark template)

#	STEP	REASON (cite in answer)

['
MISTAKE Skip \'co-prime\' in the setup; jump from p|a² to p|a without citing Theorem 1.3.
', '
FIX Always state \'co-prime\'. Always cite Theorem 1.3. Marking scheme rewards both.
']

Therefore √2 is irrational. ∎

WATCH OUT FOR

MISTAKE Skip 'co-prime' in the setup; jump from p|a² to p|a without citing Theorem 1.3.

FIX Always state 'co-prime'. Always cite Theorem 1.3. Marking scheme rewards both.

TOPIC

D

Decimal expansion of rational numbers

Every rational p/q (in lowest form) has a decimal expansion that is EITHER terminating OR non-terminating-repeating.

TOPIC

Four decimal patterns and how to identify them

TERMINATING DECIMAL

A decimal terminates (stops after finitely many digits) IFF the denominator of p/q in LOWEST FORM has prime factorisation of the form $2^m \times 5^n$ (only the primes 2 and 5). Example: 3/8 = 0.375 (terminates, denominator $8 = 2^3$). 13/3125 = 0.00416 (terminates, $3125 = 5^5$). The number of decimal places where it terminates equals

NON-TERMINATING REPEATING DECIMAL

If p/q (in lowest form) has any prime OTHER than 2 or 5 in the denominator (e.g. 3, 7, 11, 13), the decimal does NOT terminate — but it does REPEAT in a cycle. Example: 1/7 = 0.142857 142857 142857... (period 6). 1/3 = 0.333... (period 1). Non-terminating repeating decimals are still RATIONAL (this is the key insight —

NON-TERMINATING NON-REPEATING DECIMAL

Only these are IRRATIONAL. Examples: $\sqrt{2}$ = 1.41421356... (no repeating pattern), π = 3.14159265... (no repeating pattern), e = 2.71828... (no repeating pattern). The defining property: the decimal goes on forever AND never settles into a repeating cycle. CBSE 3-mark questions distinguish

REDUCE TO LOWEST FORM FIRST

The 'only 2s and 5s' rule applies to the LOWEST-FORM denominator. Without reducing, you get wrong answers. Example: 15/1600 — at face value, $1600 = 2^6 \times 5^2$, so it might appear to terminate. CORRECT: reduce 15/1600 = 3/320, then $320 = 2^6 \times 5$. Yes, still only 2s and 5s, so it terminates after MAX(6,1) =

TRY IT · SOLVE BEFORE YOU PEEK

Quick test — will p/q terminate?

Work it out before you flip the answer.

SOLUTION

1. Step 1: REDUCE p/q to lowest form. (Critical — otherwise wrong answer!)
2. Step 2: Prime-factorise the new denominator.
3. Step 3: Only 2s and 5s? → Terminates. Place count = max(power of 2, power of 5).
4. Step 4: Any other prime (3, 7, 11...)? → Non-terminating-repeating.

ANSWER Check 13/3125 . 3125 = 5^5 . Only 5s. ⇒ Terminates. Place count = max(0, 5) = 5 decimal places.

MARKS DISTRIBUTION

Marks distribution — where to spend your time

Based on 10 years of CBSE Class 10 Maths board papers (post-2020 syllabus).

10-YEAR PYQ MARKS DISTRIBUTION

FTA — HCF/LCM by prime factorisation	42%
Irrationality proofs	42%
Decimal expansions	16%
Euclid's lemma (removed in 2020)	0%

- Spend ~40% of revision time on FTA and ~40% on irrationality proofs.
- Decimal expansions = quick 1-mark MCQ; don't over-invest.
- Euclid is OUT of syllabus — skip for board, retain only if doing entrance prep.

TOPIC

E

Euclid's division lemma

Given two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, $0 \le r < b$. Used to find HCF via repeated division.

PYQ PATTERNS

Top 5 PYQ patterns to drill

#1	Prove √p irrational (3 marks)	95% of years
#2	HCF & LCM by prime factorisation + verify (3 marks)	75% of years
#3	LCM word problem (bells, tankers)	70% of years
#4	Will p/q terminate? After how many places? (1 mark)	90% of years
#5	Prove a + b√p irrational, given √p irrational (2 marks)	60% of years

TOPIC

Fundamental Theorem of Arithmetic

TRAP → TRUTH

× **MISTAKE** Take HIGHEST powers of primes for HCF (and lowest for LCM).

✓ **CORRECT** HCF = product of SMALLEST powers of COMMON primes only. LCM = product of GREATEST powers of ALL primes that appear in any factorisation. Reverse this and you fail every FTA question.

TOPIC

Word problems — HCF vs LCM

TRAP → TRUTH

× **MISTAKE** If the question asks for 'maximum / largest / greatest', use LCM. If 'minimum / smallest', use HCF.

✓ **CORRECT** Reverse: 'maximum capacity that divides exactly' → HCF; 'minimum time until events coincide again / smallest number divisible by all' → LCM. Read what is being measured: a divisor → HCF, a multiple → LCM.

TOPIC

Irrationality proof — the load-bearing assumption

TRAP → TRUTH

× **MISTAKE** Let $\sqrt{p} = a/b$ (some integers, $b \neq 0$). Then...

✓ **CORRECT** MUST state 'where a and b are CO-PRIME integers, $b \neq 0$' upfront. The contradiction at the end is that p divides BOTH a and b, contradicting their co-primeness. Without this setup, the proof is logically broken and loses 0.5 mark per the marking scheme.

TOPIC

Irrationality proof — citing Theorem 1.3

TRAP → TRUTH

× **MISTAKE** From p divides a^2, jump directly to p divides a.

✓ **CORRECT** Explicitly invoke 'By Theorem 1.3 (if a prime p divides a^2, then p divides a)'. CBSE marking scheme awards a separate mark for this citation. Omitting it costs 0.5-1 mark.

TOPIC

Decimal expansion test — must reduce first

TRAP → TRUTH

× **MISTAKE** 15/1600 has denominator 1600 = $2^6 \cdot 5^2$ — only 2s and 5s — so it terminates.

✓ **CORRECT** First REDUCE 15/1600 to lowest form: 3/320 (where 320 = $2^6 \cdot 5$). Yes, still only 2s and 5s, so it terminates. But if you don't reduce, you'll get a wrong terminating-place count. Always reduce to lowest form BEFORE inspecting the denominator's prime factorisation.

TOPIC

Non-terminating ≠ Irrational

TRAP → TRUTH

× **MISTAKE** 1/7 = 0.142857142857... is non-terminating, so it is irrational.

✓ **CORRECT** Non-terminating but REPEATING decimals are RATIONAL (they can be written as p/q). Only non-terminating AND NON-REPEATING decimals are irrational (e.g. √2, π). 1/7 is rational because it repeats.

TOPIC

Euclid's algorithm — when to stop (LEGACY)

TRAP → TRUTH

× **MISTAKE** Stop division when the quotient becomes 1 or the remainder is 'small enough'.

✓ **CORRECT** Stop ONLY when remainder = 0. HCF is the DIVISOR of that last step. Stopping early gives wrong HCF every time.

TOPPER TEMPLATE · MARK-BY-MARK

3-mark question: 'Prove that √p is irrational' (where p is a prime — typically 2, 3, 5, 7)

1	**ASSUMPTION & SETUP** 1 m	Let us assume, to the contrary, that $\sqrt{p}$ is rational. Then there exist co-prime integers a and b ($b \neq 0$) such that $\sqrt{p} = a/b$. Squaring: $a^2 = p \cdot b^2$ ... (i)
2	**APPLY THEOREM 1.3 + SUBSTITUTE** 1 m	From (i), p divides a^2. By Theorem 1.3 (if a prime p divides a^2, then p divides a), p divides a. So let $a = p \cdot c$ for some integer c. Substituting in (i): $(p \cdot c)^2 = p \cdot b^2 \Rightarrow p^2 \cdot c^2 = p \cdot b^2 \Rightarrow b^2 = p \cdot c^2$. Thus p divides b^2. Again by Theorem 1.3, p divides b.
3	**CONTRADICTION + CONCLUSION** 1 m	Therefore p is a common factor of a and b. But this contradicts our assumption that a and b are co-prime. Hence our supposition is wrong. Therefore $\sqrt{p}$ is irrational. ∎

TOPPER TEMPLATE · MARK-BY-MARK

3-mark question: 'Prove that $a + b\cdot\sqrt{p}$ is irrational, given that $\sqrt{p}$ is irrational' (chained variant)

1	**ASSUME THE OPPOSITE** 1 m	Suppose, to the contrary, that $a + b\cdot\sqrt{p}$ is rational. Call this rational number r. Then $a + b\cdot\sqrt{p} = r$.
2	**ISOLATE √P** 1 m	Rearrange: $b\cdot\sqrt{p} = r - a$, so $\sqrt{p} = (r - a) / b$. The right-hand side is a difference of rationals divided by a rational ($b \neq 0$), hence rational.
3	**CONTRADICTION + CONCLUSION** 1 m	But we are GIVEN that $\sqrt{p}$ is irrational. This is a contradiction. So our assumption is wrong. Therefore $a + b\cdot\sqrt{p}$ is irrational. ∎

TOPPER TEMPLATE · MARK-BY-MARK

3-mark word problem: bells ringing at intervals / cyclists meeting again / tankers coinciding etc.

1	**IDENTIFY OPERATION** 1 m	The events recur at intervals of a, b, c units. The next time they coincide will be at a time that is a common multiple of a, b, c. The SMALLEST such time = LCM(a, b, c).
2	**PRIME FACTORISATION + LCM** 1 m	Prime factorise: a = ..., b = ..., c = ... LCM = product of greatest powers of all primes appearing = ... units.
3	**EXPRESS THE ANSWER IN THE QUESTION'S UNITS + FINAL TIME** 1 m	Therefore the events next coincide after LCM = ... minutes (or hours). If started at, say, 6 a.m., next coincidence is at ... + start time.

PYQ PATTERNS

Top PYQ patterns to drill

#1	Prove $\sqrt{p}$ is irrational ($p \in \{2, 3, 5, 7\}$) (3 marks)	95%
#2	Find HCF and LCM by prime factorisation + verify HCF·LCM = product (3 marks)	75%
#3	LCM word problem (bells / tankers / cyclists) (3 marks)	70%
#4	Will p/q have a terminating decimal? After how many places? (1 marks)	90%
#5	Prove $a + b\cdot\sqrt{p}$ is irrational, given $\sqrt{p}$ is irrational (2 marks)	60%

MARKS DISTRIBUTION

10-year marks distribution

10-YEAR PYQ MARKS DISTRIBUTION

Fundamental Theorem of Arithmetic — HCF/LCM by prime factorisation	42%
Irrationality proofs (√2, √3, √5, mixed forms)	42%
Decimal expansions — terminating vs non-terminating-repeating	16%
Euclid's lemma + algorithm (removed from current syllabus)	0%

RECAP · MEMORISE THESE

5-line revision (memorise these)

1 FTA: every composite has a unique prime factorisation.

2 HCF = smallest powers of common primes ; LCM = greatest powers of all primes appearing.

3 HCF × LCM = product (for TWO numbers only).

4 Theorem 1.3: prime $p \mid a^2 \Rightarrow p \mid a$ (load-bearing for irrationality proofs).

5 p/q terminates iff (lowest form) denominator has only 2 and 5 as primes.

WHAT'S NEXT

Coming up next

→ Chapter 2 — Polynomials. Builds on prime factorisation idea (factorising algebraic expressions).

→ Quick check: can you state Theorem 1.3 without looking? If yes, you're ready for Polynomials.

Practise — drill, board paper, take the CBT

15-MCQ Quick Drill (20 min) → 30-mark Full Board Paper (60 min) → 20-MCQ Chapter CBT online with timer, scorecard, percentile. Find them all on the Chapter Notes page.