CHAPTER 2

Polynomials

CBSE · Mathematics · Class 10

WHAT THIS CHAPTER DOES

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's Mission

WHY THIS MATTERS

The Big Picture

TOPIC

A

Defining the Polynomial Family

TOPIC

What Counts as a Polynomial?

POINT 1	POINT 2	POINT 3	POINT 4

TOPIC

Polynomial Types by Degree

TRY IT · SOLVE BEFORE YOU PEEK

Quick Test — Is It a Polynomial?

Work it out before you flip the answer.

SOLUTION

TOPIC

Section A Recap — Tick Before Moving On

TOPIC

B

Zeroes — The Heart of the Chapter

THEOREM · LOAD-BEARING RESULT

Definition: Zero of a Polynomial

> " A real number k is a ZERO of polynomial p(x) if p(k) = 0.

STATEMENT

A real number k is a ZERO of polynomial p(x) if p(k) = 0.

WHY THIS MATTERS

- Substituting k into the expression makes it vanish
- Geometrically, the graph of $y = p(x)$ crosses or touches the x-axis at $x = k$.

WATCH OUT FOR

NOTE A polynomial of degree n has AT MOST n real zeroes — could have fewer. A parabola sitting entirely above the x-axis has zero real zeroes despite being degree 2.

WORKED EXAMPLE

Find the Zeroes of $x^2 - 5x + 6$

1. Set $p(x) = 0$: $x^2 - 5x + 6 = 0$.
2. Split the middle term: need two numbers with product 6 and sum -5 → numbers are -2 and -3.
3. Rewrite: $x^2 - 2x - 3x + 6 = 0 \Rightarrow x(x - 2) - 3(x - 2) = 0 \Rightarrow (x - 2)(x - 3) = 0$.
4. Zeroes: $x = 2$ and $x = 3$. Quick check: $p(2) = 4 - 10 + 6 = 0$ ✓; $p(3) = 9 - 15 + 6 = 0$ ✓.

TOPIC

Graphical Meaning — Three Faces of a Parabola

TOPIC

C

The Coefficient–Zero Connection (Vieta's Relations)

THEOREM · LOAD-BEARING RESULT

Quadratic: Sum & Product of Zeroes

> " If α and β are the zeroes of $p(x) = ax^2 + bx + c$ $(a \neq 0)$, then: Sum $\alpha + \beta = -b/a$ and Product $\alpha\beta = c/a$.

STATEMENT

If α and β are the zeroes of $p(x) = ax^2 + bx + c$ $(a \neq 0)$, then: Sum $\alpha + \beta = -b/a$ and Product $\alpha\beta = c/a$.

WHY THIS MATTERS

- If α, β are roots, $p(x) = a(x - \alpha)(x - \beta) = a[x^2 - (\alpha+\beta)x + \alpha\beta]$
- Matching coefficient of x: $-a(\alpha+\beta) = b \Rightarrow \alpha+\beta = -b/a$
- Matching constant: $a \cdot \alpha\beta = c \Rightarrow \alpha\beta = c/a$.

WATCH OUT FOR

NOTE The MINUS SIGN in $-b/a$ is the #1 cause of marks lost in this chapter. Sum has minus; Product does not.

WORKED EXAMPLE

Verify on $2x^2 + x - 3$

1. Compare with $ax^2 + bx + c$: $a = 2$, $b = 1$, $c = -3$.
2. Factorise: $2x^2 + x - 3 = 2x^2 + 3x - 2x - 3 = x(2x + 3) - 1(2x + 3) = (x - 1)(2x + 3)$.
3. Zeroes: $x = 1$ and $x = -3/2$.
4. Verify sum: $1 + (-3/2) = -1/2$. Formula: $-b/a = -1/2$. ✓
5. Verify product: $1 \times (-3/2) = -3/2$. Formula: $c/a = -3/2$. ✓

TRY IT · SOLVE BEFORE YOU PEEK

Quick Test — Apply Vieta's Quadratic

Work it out before you flip the answer.

SOLUTION

THEOREM · LOAD-BEARING RESULT

Cubic: Three Vieta Relations

> " If α, β, γ are the zeroes of $p(x) = ax^3 + bx^2 + cx + d$ $(a \neq 0)$: $\alpha + \beta + \gamma = -b/a$; $\alpha\beta + \beta\gamma + \gamma\alpha = +c/a$; $\alpha\beta\gamma = -d/a$.

STATEMENT

If α, β, γ are the zeroes of $p(x) = ax^3 + bx^2 + cx + d$ $(a \neq 0)$: $\alpha + \beta + \gamma = -b/a$; $\alpha\beta + \beta\gamma + \gamma\alpha = +c/a$; $\alpha\beta\gamma = -d/a$.

WHY THIS MATTERS

- Expanding $a(x - \alpha)(x - \beta)(x - \gamma)$ and matching coefficients of x^3, x^2, x, and constant gives exactly these three identities.

WATCH OUT FOR

NOTE The signs alternate: minus, plus, minus. Memorise as $-b/a$, $+c/a$, $-d/a$ in order of increasing degree of symmetric function.

WORKED EXAMPLE

Cubic Vieta Walk-Through

1. For $p(x) = 2x^3 - 5x^2 - 14x + 8$: $a = 2$, $b = -5$, $c = -14$, $d = 8$.
2. Sum of zeroes $\alpha+\beta+\gamma = -b/a = -(-5)/2 = 5/2$.
3. Sum of products $\alpha\beta+\beta\gamma+\gamma\alpha = c/a = -14/2 = -7$.
4. Product of zeroes $\alpha\beta\gamma = -d/a = -8/2 = -4$.

TOPIC

D

Constructing & Counting

WORKED EXAMPLE

Form a Quadratic from Zeroes 3 and −2

1. Sum of zeroes = $3 + (-2) = 1$.
2. Product of zeroes = $3 \times (-2) = -6$.
3. Polynomial = $k[x^2 - (\text{sum})x + \text{product}] = k[x^2 - x - 6]$.
4. Taking $k = 1$: required polynomial is $x^2 - x - 6$.
5. Note: $2x^2 - 2x - 12$, $-x^2 + x + 6$ are also valid ($k = 2$ or -1). Infinite family.

TOPPER TEMPLATE · MARK-BY-MARK

5-Mark Topper Answer — Reciprocal-Zeroes Type

1

2

3

4

MARKS DISTRIBUTION

Where the Marks Live (last 5 years)

PYQ PATTERNS

5 Patterns That Repeat Every Year

TOPPER TEMPLATE · MARK-BY-MARK

Model answer template

1 **STEP** 0.5 m

Comparing with $ax^2 + bx + c$: $a = 1$, $b = -5$, $c = 6$.

2 **STEP** 1 m

$x^2 - 5x + 6 = (x - 2)(x - 3) = 0 \Rightarrow \alpha = 2,\ \beta = 3.$

3 **STEP** 0.75 m

$\alpha + \beta = 2 + 3 = 5$. From coefficients: $-b/a = -(-5)/1 = 5$. ✓

4 **STEP** 0.75 m

$\alpha\beta = 2 \times 3 = 6$. From coefficients: $c/a = 6/1 = 6$. ✓

TOPPER TEMPLATE · MARK-BY-MARK

Model answer template

1	STEP 0.5 m	Required polynomial = $k[x^2 - (\text{sum})x + \text{product}]$, where k is any non-zero real.
2	STEP 1 m	$= k[x^2 - (-3)x + 2] = k(x^2 + 3x + 2)$.
3	STEP 0.5 m	Taking $k = 1$, the polynomial is $x^2 + 3x + 2$.

TOPPER TEMPLATE · MARK-BY-MARK

Model answer template

1	STEP 0.75 m	$6x^2 - 7x - 3$. Need two numbers whose product = $6 \times (-3) = -18$ and sum = -7: numbers -9 and $+2$.
2	STEP 0.75 m	$= 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3) = (3x + 1)(2x - 3)$.
3	STEP 0.5 m	Zeroes: $3x + 1 = 0 \Rightarrow x = -1/3$; $2x - 3 = 0 \Rightarrow x = 3/2$.
4	STEP 1 m	Sum = $-1/3 + 3/2 = (-2 + 9)/6 = 7/6 = -(-7)/6 = -b/a$. ✓ Product = $(-1/3)(3/2) = -1/2 = -3/6 = c/a$. ✓

PYQ PATTERNS

Top PYQ patterns to drill

#1	Find zeroes of given quadratic + verify sum/product (3 marks)	45%
#2	Form a quadratic from given sum & product / given zeroes (2 marks)	25%
#3	Find unknown coefficient given a condition on zeroes (reciprocal, equal magnitude opposite sign, ratio) (3 marks)	15%
#4	Geometric meaning	count zeroes from a given graph (1 marks) — 10%
#5	Cubic	apply Vieta's three relations (3 marks) — 5%

RECAP · MEMORISE THESE

Chapter 2 in One Glance

1 Polynomial = whole-number powers of x only; degree = highest power; name follows degree.

2 Zero of p(x) is any value of x that makes p(x) = 0; max real zeroes = degree.

3 Quadratic: $\alpha + \beta = -b/a$, $\alpha\beta = c/a$ (LEARN THE SIGNS).

4 Cubic: $\alpha+\beta+\gamma = -b/a$, $\alpha\beta+\beta\gamma+\gamma\alpha = c/a$, $\alpha\beta\gamma = -d/a$.

5 Form quadratic from zeroes: $k[x^2 - (\text{sum})x + \text{product}]$.

6 Graphically, zeroes are x-intercepts. Tangent to x-axis = repeated root.

WHAT'S NEXT

What's Next

→

Practice Activates Memory

readyforboards.com

Helpline: +91 70330 05444

Boards prep that builds confidence, not anxiety.