

ANSWER KEY & MARKING SCHEME · CBSE CLASS 11

Numbers, Quantification and Numerical Applications

Applied Mathematics · Chapter 1 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Short calculation (2 × 4 = 8 marks)

Q1. Convert the decimal number 45 into a binary number, showing your division steps. [2 marks]

Ans: $45 \div 2 = 22 \text{ r } 1$; $22 \div 2 = 11 \text{ r } 0$; $11 \div 2 = 5 \text{ r } 1$; $5 \div 2 = 2 \text{ r } 1$; $2 \div 2 = 1 \text{ r } 0$; $1 \div 2 = 0 \text{ r } 1$. Reading remainders bottom-to-top: $45 = 101101_2$. Check: $32+8+4+1 = 45$. ✓

Q2. Evaluate $\log 8 + \log 25 - \log 2$ (base 10), stating the laws you use. [2 marks]

Ans: By the product and quotient laws: $\log((8 \times 25)/2) = \log(200/2) = \log 100$. Since $100 = 10^2$, $\log 100 = 2$. $\log 10 = 2$ (as $\log_{10} 10 = 1$). Answer = 2.

Q3. If today is Friday, what day of the week will it be 75 days from now? Show your modular reasoning. [2 marks]

Ans: $75 = 7 \times 10 + 5$, so $75 \equiv 5 \pmod{7}$. Count 5 days forward from Friday: Sat(1), Sun(2), Mon(3), Tue(4), Wed(5). Answer = WEDNESDAY.

Q4. Given $\log 2 = 0.3010$, find $\log 5$ (base 10). [2 marks]

Ans: $\log 5 = \log(10/2) = \log 10 - \log 2 = 1 - 0.3010 = 0.6990$ (quotient law, with $\log_{10} 10 = 1$).

Section B — Word problem with full working (3 × 4 = 12 marks)

Q5. Two grades of coffee costing Rs 240/kg and Rs 360/kg are mixed to obtain a blend costing Rs 300/kg. In what ratio should the two grades be mixed? Verify your answer with the weighted-average check. [3 marks]

Ans: $C = 240$, $D = 360$, $M = 300$ ($240 < 300 < 360$ ✓). Ratio (cheaper:dearer) = $(D-M):(M-C) = (360-300):(300-240) = 60:60 = 1:1$. Check: 1 kg at 240 + 1 kg at 360 = Rs 600 for 2 kg = Rs 300/kg ✓. Answer = 1 : 1.

Q6. A boat goes 45 km downstream in 3 hours and the same distance upstream in 5 hours. Find the speed of the boat in still water and the speed of the stream. [3 marks]

Ans: Downstream = $45/3 = 15$ km/h; upstream = $45/5 = 9$ km/h. Boat speed $b = (15+9)/2 = 12$ km/h; stream speed $s = (15-9)/2 = 3$ km/h. Check: $12+3 = 15$ ✓, $12-3 = 9$ ✓.

Q7. Pipe A fills a tank in 5 hours; pipe B (a leak) empties it in 20 hours. With both open, find the time to fill the tank. [3 marks]

Ans: Rates: $A = +1/5$, leak = $-1/20$ per hour. Combined = $1/5 - 1/20 = 4/20 - 1/20 = 3/20$ per hour. Time = $1 \div (3/20) = 20/3$ hours = 6 hours 40 minutes.

Q8. Express the recurring requirement as a modulo problem: find the smallest positive integer x such that $x \equiv 4 \pmod{9}$ and $30 < x < 40$. [3 marks]

Ans: $x \equiv 4 \pmod{9}$ means $x = 4 + 9k$. For $k = 0$: 4; $k = 1$: 13; $k = 2$: 22; $k = 3$: 31; $k = 4$: 40. The only value strictly between 30 and 40 is $x = 31$. ($31 - 4 = 27 = 9 \times 3$ ✓.) Answer = 31.

Section C — Long-answer, multi-step (5 × 2 = 10 marks)

Q9. A cask is full of 50 litres of pure milk. 10 litres are drawn out and replaced with water; the same operation is then repeated a second time. (a) Find the quantity of pure milk left after the two operations. (b) Find the ratio of milk to water in the final mixture. [5 marks]

Ans: (a) Pure milk left = $V \cdot (1 - x/V)^k = 50 \cdot (1 - 10/50)^2 = 50 \cdot (1 - 1/5)^2 = 50 \cdot (4/5)^2 = 50 \cdot (16/25) = 32$ litres. (b) Total volume stays 50 L, so water = $50 - 32 = 18$ L. Milk : water = $32 : 18 = 16 : 9$.

Q10. (a) Given $\log 2 = 0.3010$ and $\log 3 = 0.4771$, evaluate $\log 6$, $\log 12$ and $\log 1.5$ (base 10). (b) Convert the binary number 110110_2 to decimal, and convert your decimal answer back to binary to verify. [5 marks]

Ans: (a) $\log 6 = \log(2 \times 3) = 0.3010 + 0.4771 = 0.7781$. $\log 12 = \log(2^2 \times 3) = 2 \log 2 + \log 3 = 2 \cdot 0.3010 + 0.4771 = 1.0791$. $\log 1.5 = \log(3/2) = \log 3 - \log 2 = 0.4771 - 0.3010 = 0.1761$. (b) $110110_2 = 32 + 16 + 0 + 4 + 2 + 0 = 54$. Back-convert 54: $54 \div 2 = 27$ r0, $27 \div 2 = 13$ r1, $13 \div 2 = 6$ r1, $6 \div 2 = 3$ r0, $3 \div 2 = 1$ r1, $1 \div 2 = 0$ r1 $\rightarrow 110110_2 \checkmark$.

★ **TOPPER ANSWER TEMPLATES**

3 TEMPLATES · MEMORISE THE FORMAT

★ **TOPPER TEMPLATE — 3 marks: 'If today is Monday, what day of the week will it be after 80 days? Show your modular reasoning.'**

Annual + SQP

Step 1 [1 mark]	Translate to modular form	A week has 7 days, so the day repeats with period 7. The day after N days is found by computing $N \bmod 7$ and counting that many days forward from today. Here we need $80 \bmod 7$.
Step 2 [1 mark]	Compute the remainder	Divide 80 by 7: $7 \times 11 = 77$, remainder = $80 - 77 = 3$. Hence $80 \equiv 3 \pmod{7}$. After 80 days the day is 3 weekdays beyond Monday. Showing the division and remainder explicitly is what earns this mark.
Step 3 [1 mark]	State the answer in words	Count 3 days forward from Monday: Tuesday (1), Wednesday (2), Thursday (3). Therefore, after 80 days, the day will be THURSDAY. (If the question asked for 80 days BEFORE Monday, count backward instead: Sunday, Saturday, Friday.)

COMMON LOSS OF MARKS:

- Computing $80 \div 7 = 11.43$ and answering '11.43 weeks' — the answer must be a named day.
- Counting from Monday as 'day 0' versus 'day 1' inconsistently — fix one convention and apply it cleanly.
- Not showing the remainder step $80 = 7 \times 11 + 3$ — examiners want the modular working visible.

★ **TOPPER TEMPLATE — 4 marks: 'A shopkeeper mixes two varieties of tea costing Rs 350/kg and Rs 500/kg to obtain a mixture costing Rs 430/kg. In what ratio should they be mixed?'**

Annual

Step 1 [1 mark]	Identify cheaper, dearer and mean	Cheaper price C = Rs 350/kg. Dearer price D = Rs 500/kg. Mean price M = Rs 430/kg. Confirm M lies between C and D: $350 < 430 < 500 \checkmark$. (If M were outside this range, no positive mixture would exist.)
Step 2 [1 mark]	Apply the alligation rule	Alligation rule: (quantity of cheaper) : (quantity of dearer) = $(D - M) : (M - C)$. Substitute: $(500 - 430) : (430 - 350) = 70 : 80 = 7 : 8$.
Step 3 [1 mark]	Verify with a weighted-average check	Take 7 kg cheaper (cost = $7 \times 350 = \text{Rs } 2,450$) + 8 kg dearer (cost = $8 \times 500 = \text{Rs } 4,000$). Total = 15 kg costing Rs 6,450. Per kg = $6,450 \div 15 = \text{Rs } 430$. \checkmark Matches the stated mean, so the ratio is correct.
Step 4 [1 mark]	State the answer with units	The two varieties must be mixed in the ratio 7 : 8 (cheaper : dearer). For every 7 kg of Rs 350 tea the shopkeeper adds 8 kg of Rs 500 tea to obtain a mixture costing Rs 430/kg.

COMMON LOSS OF MARKS:

- Putting the differences on the wrong sides — $(D - M)$ goes on the CHEAPER side, $(M - C)$ on the dearer side. The differences exchange.
- Skipping the verification step (worth 1 mark; it catches sign errors).
- Dropping units (kg) in the final ratio statement.

★ TOPPER TEMPLATE — 3 marks: 'Evaluate $\log 8 + \log 25 - \log 2$ (base 10), using the laws of logarithms.'

Almost every annual paper + SQP

Step 1 [1 mark]	Combine using the product and quotient laws	By $\log a + \log b = \log(ab)$ and $\log a - \log b = \log(a/b)$: $\log 8 + \log 25 - \log 2 = \log((8 \times 25) / 2) = \log(200 / 2) = \log 100$. State the laws being applied explicitly.
Step 2 [1 mark]	Express the argument as a power of 10	$100 = 10^2$. So $\log 100 = \log 10^2 = 2 \log 10$ (power law). Since the base is 10, $\log_{10} 10 = 1$.
Step 3 [1 mark]	State the final value	Therefore $\log 8 + \log 25 - \log 2 = 2 \times 1 = 2$. CHECK: $8 \times 25 \div 2 = 100$ and $\log_{10} 100 = 2$ because $10^2 = 100$. ✓

COMMON LOSS OF MARKS:

- Writing $\log(8 + 25 - 2)$ — addition/subtraction of arguments does NOT correspond to addition/subtraction of logs; only product/quotient does.
- Forgetting $\log_{10} 10 = 1$, leaving the answer as 'log 100' instead of the number 2.
- Not naming the law used at each step — marking schemes award method marks for stating product/quotient/power laws.

MARKING SCHEME — GENERAL NOTES

- Binary: show the division ladder (decimal→binary) or the place-value sum (binary→decimal); a back-check earns the final accuracy mark.
- Logarithms: NAME the law used at each step (product/quotient/power) — method marks are awarded for the law, not just the number. Never write $\log(a + b) = \log a + \log b$.
- Modulo: always show the remainder calculation explicitly (e.g. '75 = 7×10 + 5') before naming the day.
- Alligation: verify with the weighted-average check; it recovers the last mark on sign errors.
- Boats: state both consistency checks (down + up = 2b, down - up = 2s). Pipes: convert to RATES first, never average times; leaks are NEGATIVE rates; invert the combined rate at the end.