CHAPTER 1

Numbers, Quantification & Numerical Applications

CBSE Class 11 · Applied Mathematics (241) · Unit 1

CBSE · Applied Mathematics · Class 11

WHAT THIS CHAPTER DOES

A Convert any small number between decimal and binary, both ways, without error.

B State and apply the four laws of logarithms — and never again write log(a + b) = log a + log b.

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's mission

1. Convert any small number between decimal and binary, both ways, without error.
2. State and apply the four laws of logarithms — and never again write log(a + b) = log a + log b.
3. Translate any calendar or clock problem into 'mod 7' or 'mod 12' and solve it in two lines.
4. Solve a 4-mark alligation or boats-streams problem and verify the answer with a check.

WHY THIS MATTERS

Why this chapter matters

1. Opens Class 11 Applied Maths and carries 15 of the 80 external marks — the single heaviest unit in the course.
2. Logarithms and indices reappear in Financial Mathematics (compound interest, depreciation) and Calculus later in the year — this is where they are first built.
3. Outside the exam: binary underlies all computing, logarithms power finance/pH/decibels, and the mixture and rate techniques are everyday commerce skills.

TOPIC

A

Binary numbers

TOPIC

Representing numbers in binary and decimal

WHAT BINARY IS

Our everyday numbers use base 10 — each place is a power of ten (units, tens, hundreds). Binary uses base 2 — each place is a power of two (1, 2, 4, 8, 16, …) and only the digits 0 and 1 are allowed. This is the language of every computer, because an electronic switch has just two states: off (0) and on

BINARY → DECIMAL

To turn a binary number into decimal, multiply each digit by its place value (a power of 2) and add the results. Work right to left: the rightmost digit is the 1s place (2^0), the next is the 2s place (2^1), then 4s (2^2), 8s (2^3), and so on. Worked example: $1101_2 = 1·8 + 1·4 + 0·2 + 1·1 = 8 + 4 + 0 + 1 = 13$

DECIMAL → BINARY

To turn a decimal number into binary, divide repeatedly by 2 and record the remainder at each step; the binary number is the string of remainders read from BOTTOM to TOP. Worked example: convert 13. 13÷2 = 6 r 1; 6÷2 = 3 r 0; 3÷2 = 1 r 1; 1÷2 = 0 r 1. Reading the remainders upward gives 1101_2

WHY IT APPEARS IN APPLIED MATHS

Binary is included because Applied Mathematics aims at real-world numeracy, and the digital world runs entirely on base 2 — file sizes, memory addresses, and encryption all rest on it. In the exam this is usually an easy, guaranteed 2 marks: one conversion each way,

WORKED EXAMPLE

Convert 45 to binary, and 101101_2 to decimal

1. DECIMAL → BINARY (45): divide repeatedly by 2 and record remainders. 45÷2 = 22 r 1; 22÷2 = 11 r 0; 11÷2 = 5 r 1; 5÷2 = 2 r 1; 2÷2 = 1 r 0; 1÷2 = 0 r 1.
2. Read the remainders BOTTOM to TOP: 1, 0, 1, 1, 0, 1 → so 45 = 101101_2.
3. BINARY → DECIMAL (101101_2): weight each digit by its power of 2 (places 32, 16, 8, 4, 2, 1): 1·32 + 0·16 + 1·8 + 1·4 + 0·2 + 1·1.
4. Add: 32 + 0 + 8 + 4 + 0 + 1 = 45. ✓ The two conversions are inverses — 45 → 101101_2 → 45 confirms both are correct.
5. TAKEAWAY: decimal→binary is 'divide by 2, read remainders up'; binary→decimal is 'weight by powers of 2 and add'. Always back-check with the reverse conversion.

TOPIC

B

Indices, logarithms and antilogarithms

THEOREM · LOAD-BEARING RESULT

The laws of logarithms — the load-bearing rules

> For a base $b > 0$, $b \neq 1$, and positive arguments a, c: (product) $\log_b(ac) = \log_b a + \log_b c$; (quotient) $\log_b(a/c) = \log_b a - \log_b c$; (power) $\log_b(a^n) = n \cdot \log_b a$; (change of base) $\log_b a = (\log_k a)/(\log_k b)$. Also $\log_b 1 = 0$ and $\log_b b = 1$.

STATEMENT

A logarithm is the inverse of exponentiation: $\log_b a = x$ means $b^x = a$. The product law follows from $b^x \cdot b^y = b^{(x+y)}$; the quotient law from $b^x / b^y = b^{(x-y)}$; the power law from $(b^x)^n =$

WHY THIS MATTERS

- Logarithms turn multiplication into addition and powers into multiplication, which is why they were the basis of slide rules and log tables for centuries, and why they appear in compound-interest, pH, decibel and Richter-scale formulas
- In this chapter, log-law simplification is the single most-

WATCH OUT FOR

NOTE There is NO rule for $\log(a + b)$ — it does NOT equal $\log a + \log b$. The product law applies to a PRODUCT (ac), not a SUM. Also, log of 0 or of any negative number is UNDEFINED, because no power of a positive base is ever zero or negative.

WORKED EXAMPLE

Simplify log 8 + log 25 − log 2 (base 10)

1. QUESTION: Evaluate log 8 + log 25 − log 2 to base 10 using the laws of logarithms.
2. STEP 1 — Combine: by the product and quotient laws, log 8 + log 25 − log 2 = log((8 × 25)/2) = log(200/2) = log 100.
3. STEP 2 — Express 100 as a power of the base: 100 = 10^2, so log 100 = log 10^2 = 2·log 10 (power law).
4. STEP 3 — Since the base is 10, $\log_{10} 10 = 1$, so the value = 2 × 1 = 2.
5. CHECK: 8 × 25 ÷ 2 = 100 and 10^2 = 100, so $\log_{10} 100 = 2$. ✓
6. TAKEAWAY: always fold the expression into a single log of a product/quotient first, then reduce the argument to a power of the base. NEVER turn log 8 + log 25 into log(8 + 25).

WORKED EXAMPLE

Use indices and logs together — simplify and evaluate

1. QUESTION: (a) Simplify $2^{(3/2)} \cdot 2^{(1/2)}$. (b) If log 2 = 0.3010, find log 8 and log 5 (base 10).
2. (a) Same base, MULTIPLYING ⇒ ADD exponents: 3/2 + 1/2 = 4/2 = 2. So $2^{(3/2)} \cdot 2^{(1/2)} = 2^2 = 4$.
3. (b) $\log 8 = \log 2^3 = 3 \cdot \log 2 = 3 \times 0.3010 = 0.9030$ (power law).
4. (b) $\log 5 = \log(10/2) = \log 10 - \log 2 = 1 - 0.3010 = 0.6990$ (quotient law, with $\log_{10} 10 = 1$).
5. TAKEAWAY: writing a number as a power of a known base ($8 = 2^3$) or as a quotient ($5 = 10/2$) lets the log laws do all the work — this is exactly how the old log tables were used.

TRY IT · SOLVE BEFORE YOU PEEK

Simplify / evaluate (base 10): (a) $\log 100 + \log 1000$ (b) $\log 2 + \log 50$ (c) is $\log(3 + 7) = \log 3 + \log 7$?

SOLUTION

ANSWER (a) $\log 100 + \log 1000 = 2 + 3 = 5$. (b) $\log 2 + \log 50 = \log(2\times50) = \log 100 = 2$. (c) NO — $\log(3+7) = \log 10 = 1$, but $\log 3 + \log 7 \approx 0.477 + 0.845 = 1.322$. There is no sum law for logs.

TOPIC

C

Modulo arithmetic

THEOREM · LOAD-BEARING RESULT

Congruence modulo n — the definition behind clocks and calendars

" Two integers x and a are congruent modulo n, written $x \equiv a \pmod{n}$, if and only if $(x - a)$ is divisible by n — equivalently, x and a leave the same remainder when divided by n.

STATEMENT

$x \equiv a \pmod{n} \Leftrightarrow n \mid (x - a) \Leftrightarrow x = a + kn$ for some integer $k \Leftrightarrow (x \bmod n) = (a \bmod n)$. Modular addition and multiplication are well-defined: if $x \equiv a$ and $y \equiv b \pmod{n}$, then $x + y \equiv a + b$ and $xy \equiv ab$

WHY THIS MATTERS

- Modulo is the mathematics of CYCLIC phenomena — days of the week (mod 7), hours on a clock (mod 12 or 24), odd/even (mod 2), and divisibility tests
- Once a student sees the modulo behind these, every 'what day/hour after N steps' problem becomes a two-line solution: reduce N mod

WATCH OUT FOR

NOTE Do NOT read $x \equiv a \pmod{n}$ as 'x divided by n equals a'. The ≡ symbol means SAME REMAINDER, not division or equality. Treating ≡ as ordinary '=' is the single most common modulo error.

WORKED EXAMPLE

What day will it be after 100 days?

1. QUESTION: If today is Wednesday, what day of the week will it be after 100 days?

2. STEP 1 — Translate to mod 7: a week has 7 days, so we need 100 mod 7. Divide: $100 = 7 \times 14 + 2$, so $100 \equiv 2 \pmod 7$. After 100 days the day is TWO weekdays beyond Wednesday.

3. STEP 2 — Count two days from Wednesday: Thursday (1), Friday (2). Therefore, after 100 days, the day will be FRIDAY.

4. STEP 3 — Mental check: $7 \times 14 = 98$ days = exactly 14 weeks → same day (Wednesday). 100 days = 98 + 2 = two days beyond Wednesday = Friday. ✓

5. TAKEAWAY: for ANY 'what day after N days' problem the answer is N mod 7 days forward. The hard part is just computing the remainder cleanly — the rest is counting.

TOPIC

D

Averages, alligation and mixture

THEOREM · LOAD-BEARING RESULT

The alligation rule — the exchange of differences

> When two ingredients of cost-prices C (cheaper) and D (dearer) are mixed to produce a mixture of mean-price M with $C < M < D$, the ratio in which they must be mixed is (cheaper : dearer) = $(D – M) : (M – C)$.

STATEMENT

The rule follows from the weighted-average equation: if quantities q_1 (at C) and q_2 (at D) give mean M, then $q_1 \cdot C + q_2 \cdot D = (q_1 + q_2) \cdot M$, which rearranges to $q_1/q_2 = (D – M)/(M – C)$. The differences (D –

WHY THIS MATTERS

- Alligation is the most-asked numerical-application topic in this chapter (~22% of its marks)
- It generalises to ANY situation where two quantities at different rates combine to a known combined rate — milk-and-water, alloys, blended teas, even average-speed problems

WATCH OUT FOR

NOTE The differences EXCHANGE sides — this is the most-mistaken step. The difference (D – M), built from the dearer price, ends up on the CHEAPER side of the ratio. Drawing the criss-cross diagram every single time prevents this error. Also remember the average itself is the simple sum ÷ count only when

WORKED EXAMPLE

Tea-blending and a repeated-replacement mixture

1. QUESTION (a): A shopkeeper mixes two teas costing Rs 350/kg and Rs 500/kg to obtain a mixture costing Rs 430/kg. In what ratio?

2. (a) STEP 1 — C = 350, D = 500, M = 430. Check $350 < 430 < 500$ ✓.

3. (a) STEP 2 — Apply the rule: (cheaper : dearer) = (D – M) : (M – C) = (500 – 430) : (430 – 350) = 70 : 80 = 7 : 8.

4. (a) STEP 3 — Verify: 7 kg at 350 = Rs 2,450; 8 kg at 500 = Rs 4,000; total 15 kg = Rs 6,450; per kg = 6,450 ÷ 15 = Rs 430 ✓. Answer: 7 : 8.

5. QUESTION (b): A 40-litre cask is full of pure milk. 8 litres are drawn out and replaced by water; this is done a second time. How much pure milk remains?

6.

TOPIC

E

Time, work and distance — boats, streams and pipes

THEOREM · LOAD-BEARING RESULT

Boats and streams — the two-formula skeleton

> "If b is the speed of the boat in still water and s is the speed of the stream, then downstream speed = (b + s) and upstream speed = (b – s). Inverting, the boat's still-water speed is the AVERAGE of downstream and upstream speeds, and the stream's speed is HALF their DIFFERENCE.

STATEMENT

Definitions: downstream = (b + s); upstream = (b – s). Inverting: b = (downstream + upstream) ÷ 2; s = (downstream – upstream) ÷ 2. Consistency checks: down + up = 2b; down – up = 2s. The

WHY THIS MATTERS

- Boats-streams problems are a PYQ staple in this chapter's word-problem section
- Once the four-formula skeleton is internalised, every such problem solves in two lines: compute the two speeds from the given distances and times, then apply b = (d + u)/2 and s = (d – u)/2.

WATCH OUT FOR

NOTE Do NOT confuse the direction signs. DOWNSTREAM is FASTER (boat + stream); UPSTREAM is SLOWER (boat – stream). Reversing them is the most common mark-loss here. The English phrase fixes it: 'going with the current' = downstream = faster.

TOPIC

Pipes and cisterns — always work in RATES

TRANSLATE EVERY PIPE TO A RATE

A pipe that fills a tank in T hours has a fill-rate of 1/T of the tank per hour — this rate framing is the single most important conceptual move in the whole topic. A leak that empties a full tank in T hours has rate −1/T per hour; the minus sign is critical because a leak is mathematically just a

COMBINED RATE = SUM OF INDIVIDUAL RATES

Once pipes and leaks are in rate form, the combined rate is just the algebraic sum of the individual rates — exactly how flows add at a junction. Worked example: pipe A fills at +1/4 per hour and a leak empties at −1/12 per hour; combined rate = 1/4 − 1/12 = 3/12 −

TIME = 1 ÷ COMBINED RATE

From a combined rate of 1/6 tank per hour, the time to fill the whole tank is 1 ÷ (1/6) = 6 hours — the reciprocal of the rate. ALWAYS invert at the very end; this is the single most-forgotten step, and the reason students who set up the rates correctly still lose a mark on the final answer.

WHY AVERAGING TIMES IS WRONG

The single most common student error is averaging the individual times to find the combined time. Beginners write 'combined time = (4 + 12)/2 = 8 hours' — but this is meaningless, because times do not average for parallel processes. The correct

TOPIC

Modulo congruence

TRAP → TRUTH

× **MISTAKE** $x \equiv a \pmod{n}$ means x divided by n equals a.

✓ **CORRECT** $x \equiv a \pmod{n}$ means x AND a leave the SAME REMAINDER when divided by n — equivalently, $(x - a)$ is divisible by n. Example: $23 \equiv 3 \pmod{5}$ because $(23 - 3) = 20$ is divisible by 5, and both 23 and 3 leave remainder 3 on division by 5. Reading the ≡ symbol as ordinary '=' is the single biggest cause of wrong modulo answers.

TOPIC

Logarithm of a sum vs sum of logarithms

TRAP → TRUTH

× **MISTAKE** log(a + b) = log a + log b.

✓ **CORRECT** This is FALSE. The correct law is for a PRODUCT: log(ab) = log a + log b. There is NO simplification rule for log(a + b) — it cannot be broken up. Test it: log(2 + 8) = log 10 = 1, but log 2 + log 8 = 0.301 + 0.903 = 1.204 ≠ 1. The product becomes a sum; the sum stays locked.

TOPIC

Logarithm of zero or a negative number

TRAP → TRUTH

× **MISTAKE** log 0 = 0 and log(–5) is some negative number.

✓ **CORRECT** The logarithm of ZERO and of any NEGATIVE number is UNDEFINED in the real numbers. log a is defined only for a > 0, because a^x is always positive — no power of a positive base ever gives 0 or a negative result. Note log 1 = 0 (since $a^0 = 1$) — students confuse 'log 1 = 0' with the false 'log 0 = 0'.

TOPIC

Laws of indices — adding vs multiplying exponents

TRAP → TRUTH

× **MISTAKE** a^m × a^n = a^(m×n).

✓ **CORRECT** When MULTIPLYING powers of the same base you ADD the exponents: a^m × a^n = a^(m+n). You MULTIPLY exponents only when raising a power to a power: (a^m)^n = a^(mn). Example: $2^3 \times 2^2 = 2^5 = 32$ (add), but $(2^3)^2 = 2^6 = 64$ (multiply). Mixing these two rules is the top indices error.

TOPIC

Alligation — rule of cross multiplication

TRAP → TRUTH

× **MISTAKE** In alligation, the mixing ratio equals (cheaper price : dearer price).

✓ **CORRECT** The alligation rule is: (quantity of CHEAPER) : (quantity of DEARER) = (DEARER price – MEAN) : (MEAN – CHEAPER price), i.e. (D – M) : (M – C). The ratio is built from the DIFFERENCES with the mean, and those differences EXCHANGE sides — the dearer-side difference (D – M) lands on the cheaper side. Drawing the criss-cross diagram every time prevents this.

TOPIC

Boats and streams — direction confusion

TRAP → TRUTH

× **MISTAKE** Downstream speed = boat speed – stream speed.

✓ **CORRECT** Downstream speed = (boat speed) + (stream speed) — the stream HELPS the boat (faster). Upstream speed = (boat speed) – (stream speed) — the stream OPPOSES the boat (slower). Hence boat speed in still water = (downstream + upstream) ÷ 2 and stream speed = (downstream – upstream) ÷ 2. Swapping the signs is the most common mark-loss in this sub-topic.

TOPIC

Pipes and cisterns — leaks treated like inlets / averaging times

TRAP → TRUTH

× **MISTAKE** If pipe A fills in 6 hours and a leak empties in 12 hours, the combined fill time is (6 + 12)/2 = 9 hours.

✓ **CORRECT** Convert each pipe to a RATE: A fills at +1/6 tank per hour; the leak empties at −1/12 tank per hour. Combined rate = 1/6 − 1/12 = 1/12 tank per hour → tank fills in 12 hours. RATES add (leaks negative); TIMES do not average. Always work in rates, then invert at the end.

TOPPER TEMPLATE · MARK-BY-MARK

3 marks: 'If today is Monday, what day of the week will it be after 80 days? Show your

1 TRANSLATE TO MODULAR FORM (1 m)

A week has 7 days, so the day repeats with period 7. The day after N days is found by computing N mod 7 and counting that many days forward from today. Here we need 80 mod 7.

2 COMPUTE THE REMAINDER (1 m)

Divide 80 by 7: $7 \times 11 = 77$, remainder $= 80 - 77 = 3$. Hence $80 \equiv 3 \pmod{7}$. After 80 days the day is 3 weekdays beyond Monday. Showing the division and remainder explicitly is what earns this mark.

3 STATE THE ANSWER IN WORDS (1 m)

Count 3 days forward from Monday: Tuesday (1), Wednesday (2), Thursday (3). Therefore, after 80 days, the day will be THURSDAY. (If the question asked for 80 days BEFORE Monday, count backward instead: Sunday, Saturday, Friday.)

TOPPER TEMPLATE · MARK-BY-MARK

4 marks: 'A shopkeeper mixes two varieties of tea costing Rs 350/kg and Rs 500/kg to obtain a

1 — IDENTIFY CHEAPER, DEARER AND MEAN (1 m)

Cheaper price C = Rs 350/kg. Dearer price D = Rs 500/kg. Mean price M = Rs 430/kg. Confirm M lies between C and D: $350 < 430 < 500$ ✓. (If M were outside this range, no positive mixture would exist.)

2 — APPLY THE ALLIGATION RULE (1 m)

Alligation rule: (quantity of cheaper) : (quantity of dearer) = (D – M) : (M – C). Substitute: (500 – 430) : (430 – 350) = 70 : 80 = 7 : 8.

3 — VERIFY WITH A WEIGHTED-AVERAGE CHECK (1 m)

Take 7 kg cheaper (cost = 7 × 350 = Rs 2,450) + 8 kg dearer (cost = 8 × 500 = Rs 4,000). Total = 15 kg costing Rs 6,450. Per kg = 6,450 ÷ 15 = Rs 430. ✓ Matches the stated mean, so the ratio is correct.

4 — STATE THE ANSWER WITH UNITS (1 m)

The two varieties must be mixed in the ratio 7 : 8 (cheaper : dearer). For every 7 kg of Rs 350 tea the shopkeeper adds 8 kg of Rs 500 tea to obtain a mixture costing Rs 430/kg.

TOPPER TEMPLATE · MARK-BY-MARK

3 marks: 'Evaluate log 8 + log 25 − log 2 (base 10), using the laws of logarithms.'

1	**COMBINE USING THE PRODUCT AND QUOTIENT LAWS** 1 m	By log a + log b = log(ab) and log a − log b = log(a/b): log 8 + log 25 − log 2 = log((8 × 25) / 2) = log(200 / 2) = log 100. State the laws being applied explicitly.
2	**EXPRESS THE ARGUMENT AS A POWER OF 10** 1 m	100 = 10^2. So log 100 = log 10^2 = 2 log 10 (power law). Since the base is 10, $\log_{10}$ 10 = 1.
3	**STATE THE FINAL VALUE** 1 m	Therefore log 8 + log 25 − log 2 = 2 × 1 = 2. CHECK: 8 × 25 ÷ 2 = 100 and $\log_{10}$ 100 = 2 because 10^2 = 100. ✓

PYQ PATTERNS

Top PYQ patterns to drill

#1	Simplify a numerical expression using the laws of logarithms (e.g. evaluate log 8 + log 25 – log 2, or express $\log(x^2y/z)$ as a sum/difference). (3 marks)	Almost every annual paper + SQP
#2	Convert a decimal number to binary (or a binary number to decimal). (2 marks)	SQP + annual pattern
#3	Modulo / calendar question	find the day of the week after N days, or find x with $x \equiv a \pmod{n}$ in a given range. (2 marks) — Annual
#4	Alligation: a shopkeeper mixes two grades at given prices to hit a target price	find the ratio (or the quantity of one grade). (3 marks) — Annual

RECAP · MEMORISE THESE

Recap — the four toolkits

1 Binary — Decimal→binary: divide by 2, read remainders upward. Binary→decimal: weight each digit by its power of 2 and add. Always back-check.

2 Logarithms & indices — log(ab) = log a + log b; log(a/b) = log a – log b; log(a^n) = n·log a. NO rule for log(a+b). Indices: ADD powers when multiplying same base.

3 Modulo — x ≡ a (mod n) means (x – a) divisible by n. Day after N days = (today + N mod 7); clock hour = (now + N mod 12).

4 Alligation & boats — (cheaper : dearer) =

5 Pipes & cisterns — Work in RATES (1/T

WHAT'S NEXT

What's next

→ Unit 2 — Algebra (sets, relations and functions; sequences and series).

→ Sit the 15-MCQ Quick Drill for this chapter — under 20 minutes, target ⩾ 12/15.

→ Then the full school-pattern paper — 30 marks, internal-exam style, with full model answers.

You've mastered Unit 1 of Applied Maths.

Binary · logarithms · modulo · alligation · boats & pipes — now prove it on the paper.

readyforboards.com

Helpline: +91 70330 05444

Boards prep that builds confidence, not anxiety.