ANSWER KEY & MARKING SCHEME · CBSE CLASS 11

Algebra — Sets, Relations & Functions, Sequences & Series

Applied Mathematics · Chapter 2 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Short calculation (2 × 4 = 8 marks)

Q1. Write the power set of A = {1, 2, 3} and state how many subsets it has. [2 marks]

Ans: A has 3 elements, so the number of subsets is $2^3 = 8$. P(A) = { ∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, {1,2,3} }. The power set therefore has 8 members.

Q2. If A = {1, 2} and B = {a, b, c}, list A × B and state |A × B|. [2 marks]

Ans: A × B = {(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)}. |A × B| = |A|·|B| = 2 × 3 = 6 ordered pairs.

Q3. Find the 12th term of the AP 5, 9, 13, 17, ... [2 marks]

Ans: a = 5, d = 9 − 5 = 4. a_{12} = a + (12 − 1)d = 5 + 11 × 4 = 5 + 44 = 49.

Q4. Find the sum of the infinite GP 9, 3, 1, 1/3, ... [2 marks]

Ans: a = 9, r = 3/9 = 1/3. Since |r| = 1/3 < 1, S_∞ = a/(1 − r) = 9/(1 − 1/3) = 9/(2/3) = 9 × 3/2 = 13.5.

Section B — Word problem with full working (3 × 4 = 12 marks)

Q5. In a survey of 60 households, 35 read newspaper X and 30 read newspaper Y; 15 read both. How many read at least one of the two newspapers, and how many read neither? Verify with the four Venn regions. [3 marks]

Ans: |X| = 35, |Y| = 30, |X ∩ Y| = 15, total = 60. At least one = |X ∪ Y| = 35 + 30 − 15 = 50. Neither = 60 − 50 = 10. Check: only-X = 35 − 15 = 20, both = 15, only-Y = 30 − 15 = 15, neither = 10; 20 + 15 + 15 + 10 = 60 ✓.

Q6. Given A = {1, 2, 3, 4} and the relation R = {(1, 2), (2, 4), (3, 6), (4, 8)} (the rule y = 2x). State the domain and range of R, and explain whether R is a function. [3 marks]

Ans: Domain = set of first elements = {1, 2, 3, 4}. Range = set of second elements = {2, 4, 6, 8}. R IS a function: every input 1, 2, 3, 4 has exactly one output and no first element repeats with a different second element (it is the rule y = 2x). It is also one-one, since distinct inputs give distinct outputs.

Q7. Find the sum of the first 6 terms of the GP 3, 6, 12, 24, ... [3 marks]

Ans: a = 3, r = 6/3 = 2 (also 12/6 = 2). Since r = 2 > 1, $S_6 = a(r^6 - 1)/(r - 1) = 3(2^6 - 1)/(2 - 1)$ = 3(64 − 1)/1 = 3 × 63 = 189. Check: 3+6+12+24+48+96 = 189 ✓.

Q8. The 4th term of an AP is 14 and the 9th term is 34. Find the first term, the common difference, and the 15th term. [3 marks]

Ans: a_4 = a + 3d = 14 and a_9 = a + 8d = 34. Subtracting: 5d = 20, so d = 4. Then a = 14 − 3×4 = 2. The 15th term a_{15} = a + 14d = 2 + 14×4 = 2 + 56 = 58.

Section C — Long-answer, multi-step (5 × 2 = 10 marks)

Q9. (a) For the GP 2, 6, 18, 54, ... find the 5th term and the sum of the first 5 terms. (b) Find the sum of the infinite GP 8, 4, 2, 1, ... and state the condition under which an infinite GP has a finite sum. [5 marks]

Ans: (a) a = 2, r = 3. 5th term $a_5 = 2 \times 3^4$ = 2 × 81 = 162. Sum $S_5 = 2(3^5 - 1)/(3 - 1)$ = 2(243 − 1)/2 = 242 (check: 2+6+18+54+162 = 242 ✓). (b) For 8, 4, 2, 1, ...: a = 8, r = 1/2. Since |r| = 1/2 < 1, S_∞ = a/(1 − r) = 8/(1 − 1/2) = 16. Condition: an infinite GP has a finite sum only when |r| < 1, so that the terms shrink to zero.

Q10. **In a group of 100 students, 60 like mathematics, 50 like science and 25 like both subjects. (a) How many like at least one subject? (b) How many like only mathematics? (c) How many like only science? (d) How many like neither subject? Draw the conclusion using the inclusion-exclusion principle.** **[5 marks]**

> **Ans:** Let M = mathematics, S = science. $|M| = 60$, $|S| = 50$, $|M \cap S| = 25$, total = 100. (a) At least one = $|M \cup S| = 60 + 50 - 25 = 85$. (b) Only mathematics = $|M| - |M \cap S| = 60 - 25 = 35$. (c) Only science = $|S| - |M \cap S| = 50 - 25 = 25$. (d) Neither = $100 - 85 = 15$. Check: $35 + 25 + 25 + 15 = 100$ ✓ (only-M + both + only-S + neither = total).

★ TOPPER ANSWER TEMPLATES

3 TEMPLATES · MEMORISE THE FORMAT

★ TOPPER TEMPLATE — 3 marks: 'In a class of 40 students, 25 like cricket and 20 like football; 10 like both. How many like at least one of the two games, and how many like neither?'

Annual + SQP

Step 1 **[1 mark]**	**Write the inclusion-exclusion formula**	Let C = set who like cricket, F = set who like football. Given $|C| = 25$, $|F| = 20$, $|C \cap F| = 10$, and total = 40. The number who like AT LEAST ONE game is $|C \cup F| = |C| + |F| - |C \cap F|$. Stating this formula explicitly earns the method mark.
Step 2 **[1 mark]**	**Substitute and compute $|C \cup F|$**	$|C \cup F| = 25 + 20 - 10 = 35$. So 35 students like at least one of the two games. (Subtracting the 10 'both' avoids double-counting them.)
Step 3 **[1 mark]**	**Find 'neither' and state the answer**	Students who like NEITHER = total − $|C \cup F| = 40 - 35 = 5$. Answer: 35 like at least one game and 5 like neither. CHECK: only-cricket = 25 − 10 = 15, only-football = 20 − 10 = 10, both = 10, neither = 5; 15 + 10 + 10 + 5 = 40 ✓.

COMMON LOSS OF MARKS:

- Forgetting to subtract $|C \cap F|$ and writing $|C \cup F| = 25 + 20 = 45$ — which exceeds the class size of 40, a clear flag of double counting.
- Confusing 'at least one' (the union) with 'exactly one' (union minus intersection = 35 − 10 = 25).
- Not computing 'neither' as total − union when the question asks for it.

★ TOPPER TEMPLATE — 4 marks: 'Find the sum of the first 6 terms of the GP 2, 6, 18, ..., and also the sum of the infinite GP 8, 4, 2, 1, ...'

Annual

Step 1 **[1 mark]**	**Identify a and r for the finite GP**	For 2, 6, 18, ...: first term a = 2 and common ratio r = 6/2 = 3 (also 18/6 = 3, so it is genuinely a GP). Confirming r is the same across two consecutive ratios is what secures this mark.
Step 2 **[1 mark]**	**Apply the finite-sum formula**	Since r = 3 > 1, use $S_n = a(r^n - 1)/(r - 1)$. $S_6 = 2(3^6 - 1)/(3 - 1) = 2(729 - 1)/2 = 2 \times 728 / 2 = 728$.
Step 3 **[1 mark]**	**Identify a and r for the infinite GP and check $|r| < 1$**	For 8, 4, 2, 1, ...: a = 8 and r = 4/8 = 1/2. Since $|r| = 1/2 < 1$, the infinite sum EXISTS — state this condition explicitly; it carries a mark.
Step 4 **[1 mark]**	**Apply the infinite-sum formula and state both answers**	$S_\infty = a/(1 - r) = 8/(1 - 1/2) = 8/(1/2) = 16$. Final answers: $S_6 = 728$ (finite GP) and $S_\infty = 16$ (infinite GP). CHECK S_6: 2+6+18+54+162+486 = 728 ✓.

COMMON LOSS OF MARKS:

- Using the AP sum formula $S_n = n/2[2a + (n-1)d]$ on a GP — the two formulas are different; r is a ratio, not a difference.
- Using $S_\infty = a/(1 - r)$ without checking $|r| < 1$ (it would be wrong for $r \geq 1$).
- Arithmetic slip in computing $3^6 = 729$ — write the powers of 3 out (3, 9, 27, 81, 243, 729) to avoid it.

★ TOPPER TEMPLATE — 3 marks: 'Given A = {1, 2, 3} and the relation R = {(1, 4), (2, 5), (3, 6)} from A to B = {4, 5, 6, 7}, write the domain and range of R, and state whether R is a function.'

Annual + SQP

Step 1 **[1 mark]**	**Write the domain**	The DOMAIN of R is the set of all FIRST elements of the ordered pairs: domain = {1, 2, 3}. (These are exactly the inputs that appear.)
Step 2 **[1 mark]**	**Write the range**	The RANGE of R is the set of all SECOND elements actually achieved: range = {4, 5, 6}. Note 7 ∈ B is in the co-domain but NOT in the range, because no pair outputs 7.
Step 3 **[1 mark]**	**Decide whether R is a function and justify**	R IS a function: every input 1, 2, 3 is paired with EXACTLY ONE output (4, 5, 6 respectively) and no first element repeats with a different second element. State the one-output-per-input test as the justification.

COMMON LOSS OF MARKS:

- Swapping domain and range — domain is the FIRST elements, range the SECOND.
- Listing the whole co-domain B = {4, 5, 6, 7} as the range; the range is only the values actually achieved, so 7 is excluded.
- Saying 'function' without the justification (each input has exactly one output) — the reason carries the mark.

MARKING SCHEME — GENERAL NOTES

- Sets: power-set count is 2^n — show the count and list subsets including ∅ and the set itself; use ∈ for elements and ⊆ for subsets.
- Inclusion-exclusion: always SUBTRACT the intersection once; verify by checking the four Venn regions (only-A, both, only-B, neither) sum to the total.
- Relations/functions: domain = first elements, range = second elements achieved; justify 'function' by the one-output-per-input rule.
- AP: use $a_n = a + (n - 1)d$ and $S_n = n/2[2a + (n - 1)d]$; remember the (n – 1), not n.
- GP: $a_n = a \cdot r^{n-1}$ and $S_n = a(r^n - 1)/(r - 1)$; for the infinite sum $a/(1 - r)$, state and check $|r| < 1$ — a method mark is awarded for that check.