

CHAPTER 2

Algebra — Sets, Relations & Functions, Sequences & Series

CBSE Class 11 · Applied Mathematics (241) ·
Unit 2

CBSE · Applied Mathematics · Class 11

WHAT THIS CHAPTER DOES

A

Use set operations and the inclusion-exclusion rule to solve a Venn-diagram counting problem cleanly.

B

Tell a function from a relation, and read off the domain and range of any mapping.

Boards prep that builds confidence, not anxiety.



TODAY'S MISSION

Today's mission

- 1 Use set operations and the inclusion-exclusion rule to solve a Venn-diagram counting problem cleanly.
- 2 Tell a function from a relation, and read off the domain and range of any mapping.
- 3 Find any term or the sum of an arithmetic progression with confidence.
- 4 Sum a finite geometric progression — and an infinite one — and know exactly when the infinite sum exists.

WHY THIS MATTERS

Why this chapter matters

- 1 Unit 2 carries 15 of the 80 external marks — sequences and series alone supply nearly half of the chapter's questions.
- 2 The 'function' defined here is the central object of the Calculus unit later this year; geometric progressions are the seed of compound interest in Financial Mathematics.
- 3 Outside the exam: sets and Venn diagrams power database queries and surveys, and geometric progressions model compound growth, loans, and population — everyday commerce mathematics.

TOPIC

A

Sets — language, types and operations

TOPIC

Sets, their types and how we write them

WHAT A SET IS

A set is a well-defined collection of distinct objects, called its elements or members. 'Well-defined' means there is no ambiguity about whether an object belongs: {even numbers below 10} is a set, but {tall students} is not, because 'tall' is vague. We write sets two ways — the ROSTER form lists

TYPES OF SETS

The EMPTY (null) set \emptyset has no elements. A set is FINITE if its elements can be counted and stops, INFINITE otherwise (like the set of all natural numbers). Two sets are EQUAL if they have exactly the same elements. A is a SUBSET of B ($A \subseteq B$) if every element of A is also in B; the empty set is a subset

INTERVALS AS SETS

On the real line, an INTERVAL is the set of all real numbers between two endpoints. A CLOSED interval $[a, b]$ includes both endpoints ($a \leq x \leq b$); an OPEN interval (a, b) excludes both ($a < x < b$); half-open intervals $[a, b)$ and $(a, b]$ include one endpoint only. The bracket shape is the whole message —

OPERATIONS ON SETS

The UNION $A \cup B$ collects everything in A or B (or both). The INTERSECTION $A \cap B$ keeps only what is in BOTH. The DIFFERENCE $A - B$ keeps what is in A but not in B. The COMPLEMENT $A' = U - A$ is everything in the universal set that is NOT in A. These mirror the words 'or' 'and' 'but not'

THEOREM · LOAD-BEARING RESULT

The inclusion-exclusion principle — counting without double-counting

For any two finite sets A and B , the number of elements in their union is $|A \cup B| = |A| + |B| - |A \cap B|$. The intersection is subtracted once because its elements were counted twice — once in $|A|$ and once in $|B|$.

STATEMENT

$|A \cup B| = |A| + |B| - |A \cap B|$.
 For three sets: $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$.
 Rearranged forms used in word problems: $|A \cap B| = |A| + |B| - |A \cup B|$; number liking

WHY THIS MATTERS

- This is the most exam-tested idea in the sets section — the standard 3-4 mark survey/word problem ('how many students like at least one of cricket or football?') is solved by this single formula
- It generalises every overlapping-group count and is the backbone of probability later in the year.

WATCH OUT FOR

NOTE Do NOT forget to SUBTRACT $|A \cap B|$. Writing $|A \cup B| = |A| + |B|$ double-counts the overlap and often gives a total larger than the universal set — an instant red flag. Also keep 'at least one' (the union) distinct from 'exactly one' (union minus intersection).

WORKED EXAMPLE

A survey counting problem with Venn diagram

- 1** QUESTION: In a group of 50 people, 30 read newspaper A and 25 read newspaper B; 12 read both. How many read at least one newspaper, and how many read neither?
- 2** STEP 1 — Note the data: $|A| = 30$, $|B| = 25$, $|A \cap B| = 12$, universal total = 50.
- 3** STEP 2 — At least one = $|A \cup B| = |A| + |B| - |A \cap B| = 30 + 25 - 12 = 43$.
- 4** STEP 3 — Neither = total - $|A \cup B| = 50 - 43 = 7$.
- 5** CHECK with the four Venn regions: only-A = $30 - 12 = 18$, both = 12, only-B = $25 - 12 = 13$, neither = 7; $18 + 12 + 13 + 7 = 50 \checkmark$.
- 6** TAKEAWAY: subtract the overlap once to count the union, then 'neither' is total minus union. Drawing and filling the four regions back-checks every answer.

TOPIC

B

Relations and functions

TOPIC

Cartesian product, relations and functions

ORDERED PAIRS & CARTESIAN PRODUCT

An ORDERED PAIR (a, b) has a fixed first and second entry, so $(1, 2) \neq (2, 1)$. The CARTESIAN PRODUCT $A \times B$ is the set of ALL ordered pairs with first element from A and second from B : $A \times B = \{(a, b) : a \in A, b \in B\}$. If $A = \{1, 2\}$ and $B = \{x, y\}$ then $A \times B = \{(1, x), (1, y), (2, x), (2, y)\}$ — four pairs.

RELATIONS & THEIR DOMAIN/RANGE

A RELATION R from A to B is any subset of $A \times B$ — that is, any chosen collection of ordered pairs. The DOMAIN of R is the set of all FIRST elements appearing in its pairs (the inputs), and the RANGE is the set of all SECOND elements actually achieved (the outputs). The set B itself

WHAT MAKES A RELATION A FUNCTION

A FUNCTION is a special relation in which EVERY element of the domain is paired with EXACTLY ONE element of the co-domain — one input, one output, no exceptions. Equivalently, no first element may appear twice with different second elements. So $\{(1, 4), (2, 5), (3, 6)\}$ is a

TYPES OF FUNCTIONS

A function is ONE-ONE (injective) if different inputs always give different outputs — no two arrows land on the same target. It is MANY-ONE if at least two inputs share an output. It is ONTO (surjective) if every element of the co-domain is hit by some input, so the range equals the co-domain.

WORKED EXAMPLE

Cartesian product, domain, range and the function test

- 1** QUESTION: $A = \{1, 2, 3\}$, $B = \{4, 5\}$. (a) Write $A \times B$ and state $|A \times B|$. (b) For $R = \{(1, 4), (2, 5), (3, 4)\}$, give the domain and range and say whether R is a function.

- 2** (a) $A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$ — every first from A paired with every second from B . $|A \times B| = |A| \cdot |B| = 3 \times 2 = 6$ ✓ (six pairs listed).

- 3** (b) DOMAIN of $R =$ first elements $= \{1, 2, 3\}$. RANGE of $R =$ second elements achieved $= \{4, 5\}$ (4 appears twice but is listed once).

- 4** (b) Is R a function? Each input 1, 2, 3 has EXACTLY ONE output (4, 5, 4 respectively); no first element repeats with a different second element. So YES, R is a function — and it is many-one, since inputs 1 and 3 both give output 4.

- 5** TAKEAWAY: the Cartesian product is every possible pairing; a relation picks some of them; it is a function only if no input is sent to two different outputs.

TOPIC

C

Sequences and series — arithmetic progressions

THEOREM · LOAD-BEARING RESULT

Arithmetic progression — equal steps

“ An arithmetic progression (AP) is a sequence in which each term after the first is obtained by **ADDING** a fixed number d (the common difference) to the previous term. Its n th term is $a_n = a + (n - 1)d$, and the sum of the first n terms is $S_n = n/2 [2a + (n - 1)d] = n/2 (a + l)$, where a is the first term and l the last term.

STATEMENT

AP terms: $a, a + d, a + 2d, \dots, a + (n - 1)d$. n th term: $a_n = a + (n - 1)d$. Sum: $S_n = n/2 [2a + (n - 1)d]$. When the last term $l = a_n$ is known, the sum simplifies to $S_n = n/2 (a + l)$ — literally 'number of terms

WHY THIS MATTERS

- APs model anything that grows by a **CONSTANT** amount each period — a fixed monthly saving, equal annual increments, seats increasing row by row in an auditorium
- They were introduced in Class 10 and are reused here directly, then extended to geometric progressions, so a secure grip on

WATCH OUT FOR

NOTE Use $n - 1$, not n , in the n th-term formula — the first term already 'uses up' zero steps of d . A frequent slip is computing $a + nd$ instead of $a + (n - 1)d$. Also keep the two sum forms straight: use $n/2[2a + (n-1)d]$ when you know d , and $n/2(a + l)$ when you know the last term.

WORKED EXAMPLE

AP — find the 10th term and the sum of 10 terms

- 1 QUESTION: For the AP 3, 7, 11, 15, ..., find (a) the 10th term and (b) the sum of the first 10 terms.

- 2 STEP 1 — Identify a and d: first term $a = 3$; common difference $d = 7 - 3 = 4$ (also $11 - 7 = 4$, confirming an AP).

- 3 (a) 10th term: $a_{10} = a + (10 - 1)d = 3 + 9 \times 4 = 3 + 36 = 39$.

- 4 (b) Sum of 10 terms: $S_{10} = n/2 [2a + (n - 1)d] = 10/2 [2 \times 3 + 9 \times 4] = 5 \times [6 + 36] = 5 \times 42 = 210$.

- 5 CHECK with the average form: $S_{10} = n/2 (a + l) = 10/2 \times (3 + 39) = 5 \times 42 = 210 \checkmark$ — both sum formulas agree.

- 6 TAKEAWAY: pin down a and d first; then the nth-term and the sum are one substitution each. Always remember the $(n - 1)$.

TOPIC

D

Sequences and series — geometric progressions

THEOREM · LOAD-BEARING RESULT

Geometric progression — equal multipliers

“ A geometric progression (GP) is a sequence in which each term after the first is obtained by **MULTIPLYING** the previous term by a fixed non-zero number r (the common ratio). Its n th term is $a_n = a \cdot r^{n-1}$; the sum of the first n terms is $S_n = a(r^n - 1)/(r - 1)$ for $r \neq 1$; and the sum of an **INFINITE** GP is $S_\infty = a/(1 - r)$, valid only when $|r| < 1$.

STATEMENT

GP terms: $a, ar, ar^2, \dots, ar^{n-1}$.
 n th term: $a_n = a \cdot r^{n-1}$. Finite sum: $S_n = a(r^n - 1)/(r - 1)$ (equivalently $a(1 - r^n)/(1 - r)$, handier when $r < 1$). Infinite sum ($|r| < 1$): $S_\infty = a/(1 - r)$, because $r^n \rightarrow 0$ as $n \rightarrow \infty$. The

WHY THIS MATTERS

- GPs model **CONSTANT-FACTOR** growth or decay — compound interest, depreciation, population growth, radioactive decay, and the bouncing-ball problem
- The infinite-sum result is striking and exam-favourite: an unending sum like $8 + 4 + 2 + 1 + \dots$ totals a finite 16.

WATCH OUT FOR

NOTE In a GP you **MULTIPLY** by r — do not add it like an AP's d , and do not use the AP sum formula. For the infinite sum, **ALWAYS** check $|r| < 1$ first; if $|r| \geq 1$ the series has **NO** finite sum. State the condition explicitly to earn the method mark.

WORKED EXAMPLE

GP — 5th term, finite sum, and an infinite sum

1 QUESTION: (a) For the GP 2, 6, 18, 54, ..., find the 5th term and the sum of the first 5 terms. (b) Find the sum of the infinite GP 8, 4, 2, 1, ...

2 (a) STEP 1 — $a = 2$, $r = 6/2 = 3$ (also $18/6 = 3$, so it is a GP). 5th term: $a_5 = a \cdot r^4 = 2 \times 3^4 = 2 \times 81 = 162$.

3 (a) STEP 2 — Since $r = 3 > 1$: $S_5 = a(r^5 - 1)/(r - 1) = 2(3^5 - 1)/(3 - 1) = 2(243 - 1)/2 = 2 \times 242 / 2 = 242$.

4 (a) CHECK: $2 + 6 + 18 + 54 + 162 = 242 \checkmark$.

5 (b) For 8, 4, 2, 1, ...: $a = 8$, $r = 4/8 = 1/2$. Since $|r| = 1/2 < 1$, the infinite sum exists: $S_\infty = a/(1 - r) = 8/(1 - 1/2) = 8/(1/2) = 16$.

6 TAKEAWAY: GP multiplies by r each step; the finite sum uses $a(r^n - 1)/(r - 1)$, and the infinite sum $a/(1 - r)$ — but only after confirming $|r| < 1$.

TRY IT · SOLVE BEFORE YOU PEEK

(a) Find the 7th term of the AP 5, 8, 11, ... (b) Sum the infinite GP 9, 3, 1, 1/3, ... (c) Does the infinite GP 1, 2, 4, 8, ... have a finite sum?

SOLUTION

ANSWER (a) $a = 5, d = 3; a_7 = 5 + 6 \times 3 = 23$. (b) $a = 9, r = 1/3, |r| < 1$, so $S_\infty = 9/(1 - 1/3) = 9/(2/3) = 13.5$. (c) NO — here $r = 2$, and $|r| = 2 \geq 1$, so the terms grow and the infinite series diverges; it has no finite sum.

TOPIC

Power set — number of subsets

TRAP → TRUTH

× **MISTAKE** A set with n elements has n subsets (or $2n$ subsets).

✓ **CORRECT** A set with n elements has exactly 2^n subsets, and its power set $P(A)$ — the set of ALL these subsets — therefore has 2^n members. Example: $A = \{1, 2, 3\}$ has $2^3 = 8$ subsets, namely $\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$. Both the empty set \emptyset and the set A itself are always counted. Writing 'n subsets' or '2n subsets' instead of 2^n is the commonest sets error.

TOPIC

Element-of (\in) versus subset-of (\subset)

TRAP → TRUTH

× **MISTAKE** Writing $\{2\} \in A$ or $2 \subset A$ interchangeably.

✓ **CORRECT** Use \in between an ELEMENT and a set: $2 \in \{1, 2, 3\}$. Use \subset (or \subseteq) between a SET and a set: $\{2\} \subset \{1, 2, 3\}$. So $2 \in A$ but $\{2\} \subseteq A$ — the single element 2 belongs to A, while the SET $\{2\}$ is a subset of A. Mixing the two symbols loses easy marks; check both sides of the symbol: is the left side a single element or a whole set?

TOPIC

Cartesian product is ordered

TRAP → TRUTH

× **MISTAKE** $A \times B = B \times A$ (the order of the two sets does not matter).

✓ **CORRECT** $A \times B$ is generally NOT equal to $B \times A$, because the pairs are ORDERED: $(a, b) \neq (b, a)$ unless $a = b$. Example: $A = \{1, 2\}$, $B = \{x\}$. Then $A \times B = \{(1, x), (2, x)\}$ but $B \times A = \{(x, 1), (x, 2)\}$ — different pairs. Only the COUNT matches: $|A \times B| = |B \times A| = |A| \cdot |B|$. The sets themselves are equal only when $A = B$.

TOPIC

Every relation is a function

TRAP → TRUTH

× **MISTAKE** Any set of ordered pairs (any relation) is a function.

✓ **CORRECT** A function is a SPECIAL relation in which EVERY input (first element) is paired with EXACTLY ONE output (second element). If a single x-value maps to two different y-values, it is a relation but NOT a function. Example: $\{(1, 2), (1, 3)\}$ is a relation but not a function (input 1 has two outputs). Test: no x repeats with a different y — equivalently, the vertical-line test for graphs.

TOPIC

Domain versus range

TRAP → TRUTH

× **MISTAKE** The domain is the set of second elements (outputs) and the range is the first elements (inputs).

✓ **CORRECT** It is the OTHER way round. DOMAIN = the set of all FIRST elements (inputs / x-values). RANGE = the set of all SECOND elements (outputs / y-values) that are actually achieved. Memory hook: 'domain comes first, range comes after' — Domain → Dependent. For $\{(1, 4), (2, 5), (3, 6)\}$: domain = $\{1, 2, 3\}$, range = $\{4, 5, 6\}$.

TOPIC

Geometric progression — adding instead of multiplying / sum formula

TRAP → TRUTH

× **MISTAKE** In a GP you ADD the common ratio to get the next term, and $S_n = n/2[2a + (n-1)r]$.

✓ **CORRECT** In a GP you MULTIPLY by the common ratio r to get the next term (you ADD the common difference only in an AP). The GP n th term is $a_n = a \cdot r^{n-1}$, and the finite sum is $S_n = a(r^n - 1)/(r - 1)$ for $r \neq 1$ — NOT the AP sum formula. Mixing the AP and GP formulas, or treating r as if it were d , is the top sequences-and-series error.

TOPIC

Infinite GP sum exists for every ratio

TRAP → TRUTH

× **MISTAKE** Every infinite geometric series has a finite sum given by $a/(1 - r)$.

✓ **CORRECT** The infinite-GP sum $S_{\infty} = a/(1 - r)$ is valid ONLY when $|r| < 1$ (the terms shrink towards zero). If $|r| \geq 1$ the terms do NOT shrink, the partial sums grow without bound, and the infinite series has NO finite sum (it diverges). Always state and check the condition $|r| < 1$ before using the formula — examiners award a mark for that check and deduct for using it blindly.

TOPPER TEMPLATE · MARK-BY-MARK

3 marks: 'In a class of 40 students, 25 like cricket and 20 like football; 10 like both. How

- 1** **WRITE THE INCLUSION-EXCLUSION FORMULA**
1 m

Let C = set who like cricket, F = set who like football. Given $|C| = 25$, $|F| = 20$, $|C \cap F| = 10$, and total = 40. The number who like AT LEAST ONE game is $|C \cup F| = |C| + |F| - |C \cap F|$. Stating this formula explicitly earns the method mark.
- 2** **SUBSTITUTE AND COMPUTE $|C \cup F|$**
1 m

$|C \cup F| = 25 + 20 - 10 = 35$. So 35 students like at least one of the two games. (Subtracting the 10 'both' avoids double-counting them.)
- 3** **FIND 'NEITHER' AND STATE THE ANSWER**
1 m

Students who like NEITHER = total - $|C \cup F| = 40 - 35 = 5$. Answer: 35 like at least one game and 5 like neither. CHECK: only-cricket = $25 - 10 = 15$, only-football = $20 - 10 = 10$, both = 10, neither = 5; $15 + 10 + 5 = 40 \checkmark$.

TOPPER TEMPLATE · MARK-BY-MARK

4 marks: 'Find the sum of the first 6 terms of the GP 2, 6, 18, ..., and also the sum of the

1 IDENTIFY A AND R FOR THE FINITE GP

1 m

For 2, 6, 18, ...: first term $a = 2$ and common ratio $r = 6/2 = 3$ (also $18/6 = 3$, so it is genuinely a GP). Confirming r is the same across two consecutive ratios is what secures this mark.

2 APPLY THE FINITE-SUM FORMULA

1 m

Since $r = 3 > 1$, use $S_n = a(r^n - 1)/(r - 1)$. $S_6 = 2(3^6 - 1)/(3 - 1) = 2(729 - 1)/2 = 2 \times 728 / 2 = 728$.

3 IDENTIFY A AND R FOR THE INFINITE GP AND CHECK $|R| < 1$

1 m

For 8, 4, 2, 1, ...: $a = 8$ and $r = 4/8 = 1/2$. Since $|r| = 1/2 < 1$, the infinite sum EXISTS — state this condition explicitly; it carries a mark.

4 APPLY THE INFINITE-SUM FORMULA AND STATE BOTH ANSWERS

1 m

$S_\infty = a/(1 - r) = 8/(1 - 1/2) = 8/(1/2) = 16$. Final answers: $S_6 = 728$ (finite GP) and $S_\infty = 16$ (infinite GP). CHECK S_6 : $2+6+18+54+162+486 = 728 \checkmark$.

TOPPER TEMPLATE · MARK-BY-MARK

3 marks: 'Given $A = \{1, 2, 3\}$ and the relation $R = \{(1, 4), (2, 5), (3, 6)\}$ from A to $B = \{4, 5, 6, 7\}$.

- 1** **WRITE THE DOMAIN**
1 m

The DOMAIN of R is the set of all FIRST elements of the ordered pairs: domain = $\{1, 2, 3\}$. (These are exactly the inputs that appear.)
- 2** **WRITE THE RANGE**
1 m

The RANGE of R is the set of all SECOND elements actually achieved: range = $\{4, 5, 6\}$. Note $7 \in B$ is in the co-domain but NOT in the range, because no pair outputs 7.
- 3** **DECIDE WHETHER R IS A FUNCTION AND JUSTIFY**
1 m

R IS a function: every input 1, 2, 3 is paired with EXACTLY ONE output (4, 5, 6 respectively) and no first element repeats with a different second element. State the one-output-per-input test as the justification.

PYQ PATTERNS

Top PYQ patterns to drill

#1	Find a specified term or the sum of n terms of an AP or GP from the given first term and common difference / ratio. (3 marks)	Almost every annual paper + SQP
#2	Sum an infinite geometric progression ($ r < 1$) using $S_{\infty} = a/(1 - r)$, e.g. a recurring-decimal or a 'bouncing ball' set-up. (3 marks)	SQP + annual pattern
#3	Venn-diagram word problem	given $ A $, $ B $ and $ A \cup B $ (or survey counts), find $ A \cap B $ or the 'only-A' / 'only-B' counts. (3-4 marks) — Annual
#4	Write the Cartesian product $A \times B$, or state the domain and range of a given relation/ function, and decide if it is a function. (2-3 marks)	Annual
#5		power set count

RECAP · MEMORISE THESE

Recap — the three toolkits

1 Sets — \in for element, \subseteq for subset; power set of n elements has 2^n members. Operations: \cup (or), \cap (and), $-$ (but not), A' (not). Counting: $|A \cup B| = |A| + |B| - |A \cap B|$.

2 Relations — $A \times B =$ ordered pairs, $|A \times B| = |A| \cdot |B|$, and $A \times B \neq B \times A$ in general. A relation is any subset of $A \times B$; domain = first elements, range = second elements achieved.

3 Functions — A relation where every input has EXACTLY ONE output (vertical-line test). Types: one-one (injective), many-one, onto (surjective), into.

4 Arithmetic progression — Add d

5 Geometric progression —

WHAT'S NEXT

What's next



- Unit 3 — Mathematical and Logical Reasoning (statements, logical connectives, validity).
- Sit the 15-MCQ Quick Drill for this chapter — under 20 minutes, target $\geq 12/15$.
- Then the full school-pattern paper — 30 marks, internal-exam style, with full model answers.

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