ANSWER KEY & MARKING SCHEME · CBSE CLASS 11

Mathematical Reasoning

Applied Mathematics · Chapter 3 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Short answer (2 × 4 = 8 marks)

Q1. State, with reason, whether each is a statement and give its truth value: (a) 'The number 17 is prime.' (b) 'Solve the equation $x + 1 = 0$.' [2 marks]

Ans: (a) STATEMENT — declarative with a fixed truth value; TRUE (17 has no factors except 1 and 17). (b) NOT a statement — it is a command, so no truth value can be assigned.

Q2. Write the negation of: (a) 'All triangles have three sides.' (b) 'Some integers are negative.' [2 marks]

Ans: (a) Negation of a universal: 'Some triangle does NOT have three sides' (there exists a triangle without three sides). (b) Negation of an existential: 'No integer is negative' (all integers are not negative).

Q3. Let p: '5 is odd' (T) and q: '5 is even' (F). Find the truth value of (i) $p \wedge q$ and (ii) $p \vee q$, stating the rule used. [2 marks]

Ans: (i) $p \wedge q$ is true only if both are true; q is false, so $p \wedge q$ is FALSE. (ii) $p \vee q$ (inclusive) is true if at least one is true; p is true, so $p \vee q$ is TRUE.

Q4. Rewrite 'A number is a multiple of 9 only if it is a multiple of 3' in 'if ... then ...' form, and identify the necessary condition. [2 marks]

Ans: 'p only if q' means $p \Rightarrow q$. So: 'IF a number is a multiple of 9, THEN it is a multiple of 3.' The necessary condition (q) is 'the number is a multiple of 3'.

Section B — Reasoning with full working (3 × 4 = 12 marks)

Q5. For the statement 'If a number is divisible by 10, then it is divisible by 5', write (i) the converse, (ii) the inverse and (iii) the contrapositive. State which one is logically equivalent to the original. [3 marks]

Ans: Let p: 'a number is divisible by 10', q: 'it is divisible by 5'. (i) Converse ($q \Rightarrow p$): 'If a number is divisible by 5, then it is divisible by 10.' (ii) Inverse ($\sim p \Rightarrow \sim q$): 'If a number is not divisible by 10, then it is not divisible by 5.' (iii) Contrapositive ($\sim q \Rightarrow \sim p$): 'If a number is not divisible by 5, then it is not divisible by 10.' The CONTRAPOSITIVE is logically equivalent to the original.

Q6. Check whether the statement 'For every real number x, $x^2 \geqslant 0$' is true, justifying your answer; then state whether a single example would be enough to prove it. [3 marks]

Ans: The statement is TRUE: for any real x, the square x^2 is the product of two equal-sign factors, so x^2 is never negative — $x^2 > 0$ for $x \neq 0$ and $x^2 = 0$ for $x = 0$, hence $x^2 \geqslant 0$ always. A single example is NOT enough to PROVE a universal ('for every') statement; only a general argument (as given) proves it, though a single counter-example could disprove one.

Q7. Disprove the statement: 'The product of two irrational numbers is always irrational.' [3 marks]

Ans: This is a universal claim, so one counter-example disproves it. Take the irrational numbers $\sqrt{2}$ and $\sqrt{2}$. Their product is $\sqrt{2} \times \sqrt{2} = 2$, which is RATIONAL. Hence there exist two irrational numbers whose product is rational, so the statement is FALSE.

Q8. Identify the component statements and the connective in 'The number 12 is divisible by 3 and by 4', and find the truth value of the compound statement. [3 marks]

Ans: Components: r = '12 is divisible by 3' (TRUE, $12 \div 3 = 4$) and s = '12 is divisible by 4' (TRUE, $12 \div 4 = 3$). Connective: AND (conjunction). Since both components are true, the compound statement $r \wedge s$ is TRUE.

Section C — Long-answer, multi-step (5 × 2 = 10 marks)

Q9. (a) Write the negation of each: (i) 'Every student in the class likes mathematics.' (ii) 'There exists a positive integer that is the largest.' (b) For the conditional 'If two lines are parallel, then they do not intersect', write the contrapositive and state whether the original and the contrapositive have the same truth value. **[5 marks]**

> **Ans:** (a)(i) Negation: 'Some student in the class does NOT like mathematics' (there exists a student who does not like mathematics). (a)(ii) Negation: 'For every positive integer, it is NOT the largest' (no positive integer is the largest) — this negation is true, since there is no largest positive integer. (b) Let p: 'two lines are parallel', q: 'they do not intersect'. Contrapositive ($\sim q \Rightarrow \sim p$): 'If two lines DO intersect, then they are NOT parallel.' The original and its contrapositive are logically equivalent, so they ALWAYS have the SAME truth value (both true here).

Q10. (a) Validate by the direct method: 'If n is an even integer, then n^2 is even.' (b) Validate by the method of contrapositive: 'If n^2 is even, then n is even.' **[5 marks]**

> **Ans:** (a) Direct: assume n is even, so $n = 2k$ for some integer k. Then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$, which is of the form 2m — even. Hence the statement is TRUE. (b) Contrapositive: the statement 'if n^2 is even then n is even' is equivalent to 'if n is NOT even (odd) then n^2 is NOT even (odd)'. Assume n is odd, $n = 2k + 1$; then $n^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$, which is odd. So whenever n is odd, n^2 is odd — the contrapositive holds, and therefore the original statement 'if n^2 is even then n is even' is TRUE.

★ TOPPER ANSWER TEMPLATES

3 TEMPLATES · MEMORISE THE FORMAT

★ TOPPER TEMPLATE — 3 marks: 'Write the converse, inverse and contrapositive of the statement: "If a number is divisible by 6, then it is divisible by 3."'

Almost every annual paper + SQP

Step 1 [1 mark]	**Identify p and q clearly**	Let p: 'a number is divisible by 6' and q: 'the number is divisible by 3'. The given statement is the conditional $p \Rightarrow q$ ('if p then q'). Naming p and q first prevents swapping errors in the next step.
Step 2 [1 mark]	**Write the converse and the inverse**	CONVERSE (swap, $q \Rightarrow p$): 'If a number is divisible by 3, then it is divisible by 6.' INVERSE (negate both, $\sim p \Rightarrow \sim q$): 'If a number is NOT divisible by 6, then it is NOT divisible by 3.'
Step 3 [1 mark]	**Write the contrapositive and note the equivalence**	CONTRAPOSITIVE (negate and swap, $\sim q \Rightarrow \sim p$): 'If a number is NOT divisible by 3, then it is NOT divisible by 6.' Remark: the contrapositive is logically equivalent to the original (both true here); the converse is NOT (e.g. 9 is divisible by 3 but not by 6, so the converse is false).

COMMON LOSS OF MARKS:

- Giving the converse when asked for the contrapositive (or vice-versa) — converse only swaps; contrapositive negates AND swaps.
- Negating only one of p, q in the inverse instead of both.
- Not naming p and q at the start, which makes the three forms hard for the examiner to follow.

★ TOPPER TEMPLATE — 2 marks: 'Write the negation of each statement: (a) "All teachers are punctual." (b) "Some students like mathematics."'

Annual + SQP

Step 1 [1 mark]	**Negate the universal statement**	(a) The statement is universal ('all'). Its negation is existential-of-the-opposite: 'There exists at least one teacher who is NOT punctual', equivalently 'Some teachers are not punctual.' (NOT 'all teachers are not punctual' — one exception is enough to make 'all are punctual' false.)
Step 2 [1 mark]	**Negate the existential statement**	(b) The statement is existential ('some'). Its negation is universal-of-the-opposite: 'NO student likes mathematics', equivalently 'All students do NOT like mathematics.' Rule: negation flips $\forall$ to $\exists$-not and $\exists$ to $\forall$-not.

COMMON LOSS OF MARKS:

- Writing 'All teachers are not punctual' as the negation of 'All teachers are punctual' — the correct negation is 'some are not'.
- Forgetting that negating 'some ... do' gives 'none ... do' (a universal), not 'some ... do not'.
- Adding a double negative that accidentally restates the original.

★ TOPPER TEMPLATE — 3 marks: 'Check whether the statement is true: "For every real number x, x^2 is greater than x." If false, disprove it.'

Most years

Step 1 **[1 mark]**	**Classify the statement and choose a method**	The statement is UNIVERSAL: 'for every real number x, $x^2 > x$'. To DISPROVE a universal statement it is enough to produce ONE counter-example — a single x for which $x^2 > x$ fails. (To prove it we would need all x; to disprove, one suffices.)
Step 2 **[1 mark]**	**Produce a counter-example**	Take $x = 1/2$. Then $x^2 = 1/4 = 0.25$, and $x = 0.5$. Here $x^2 = 0.25$ is NOT greater than $x = 0.5$ — in fact $0.25 < 0.5$. So the statement fails for $x = 1/2$. ($x = 1$ also works: $1^2 = 1$ is not greater than 1.)
Step 3 **[1 mark]**	**State the conclusion**	Since there exists a real number ($x = 1/2$) for which $x^2 > x$ is false, the universal statement 'for every real x, $x^2 > x$' is FALSE. One counter-example is sufficient to disprove it.

COMMON LOSS OF MARKS:

- Trying to 'prove' a false universal statement by listing examples where it happens to hold — examples cannot prove a universal, and one counter-example disproves it.
- Choosing a counter-example that does not actually fail the condition (always verify the arithmetic).
- Not stating the final true/false conclusion explicitly.

MARKING SCHEME — GENERAL NOTES

- Statements: award the first mark for correctly classifying statement vs non-statement (declarative + fixed truth value); the second for the correct truth value with a one-line reason. Questions, commands, opinions and open sentences are NOT statements.
- Negation: for 'all' write 'some ... not' ($\forall \to \exists$-not); for 'some' write 'none / all ... not' ($\exists \to \forall$-not). Merely flipping the verb while keeping the quantifier earns no mark.
- Converse / inverse / contrapositive: converse = swap ($q \Rightarrow p$); inverse = negate both ($\sim p \Rightarrow \sim q$); contrapositive = negate AND swap ($\sim q \Rightarrow \sim p$). Only the contrapositive is equivalent to the original — state this for the equivalence mark.
- AND / OR: AND true only if BOTH true; inclusive OR false only if BOTH false. Always identify components + connective first.
- Validation: a universal is disproved by ONE counter-example and proved only by a general argument; name the method (direct / contrapositive / contradiction) and state the final true/false conclusion explicitly.