ANSWER KEY & MARKING SCHEME · CBSE CLASS 11

Calculus

Applied Mathematics · Chapter 4 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Short calculation (2 × 4 = 8 marks)

Q1. Differentiate $f(x) = x^2$ from first principles. [2 marks]

Ans: $f'(x) = \lim_{h\to0}[(x+h)^2 - x^2]/h = \lim_{h\to0}[(2xh + h^2)/h] = \lim_{h\to0}(2x + h) = 2x$.

Q2. Evaluate $\lim_{x\to2} (x^2 - 4)/(x - 2)$, justifying the simplification. [2 marks]

Ans: Substitution gives 0/0 (indeterminate). Factor: (x–4) numerator = (x–2)(x+2); cancel (x–2): the expression = (x+2). Then $\lim_{x\to2}(x+2) = 4$.

Q3. Find the domain of $f(x) = \sqrt{(x - 3)}$. [2 marks]

Ans: The expression under the square root must be ⩾ 0: $x - 3 \geqslant 0 \Rightarrow x \geqslant 3$. Domain = $\{x : x \geqslant 3\}$.

Q4. Differentiate $y = (2x + 1)^5$ with respect to x. [2 marks]

Ans: Chain rule: $dy/dx = 5(2x + 1)^4 \cdot d/dx(2x + 1) = 5(2x + 1)^4 \cdot 2 = 10(2x + 1)^4$.

Section B — Application with full working (3 × 4 = 12 marks)

Q5. Differentiate $y = (3x^2)/(x + 1)$ with respect to x, using the quotient rule. [3 marks]

Ans: $u = 3x^2$ ($u' = 6x$), $v = x + 1$ ($v' = 1$). Quotient rule: $y' = (u'v - uv')/v^2 = (6x(x+1) - 3x^2\cdot1)/(x+1)^2 = (6x^2 + 6x - 3x^2)/(x+1)^2 = (3x^2 + 6x)/(x+1)^2$.

Q6. The total cost of producing x units is $C(x) = 0.01x^2 + 4x + 100$. Find the marginal cost function and evaluate it at x = 100. Interpret the result. [3 marks]

Ans: $MC = dC/dx = 0.02x + 4$. At x = 100: $MC = 0.02\cdot100 + 4 = 2 + 4 = 6$. Interpretation: when 100 units are already being made, the 101st unit costs approximately Rs 6.

Q7. Find the interval on which $f(x) = x^2 - 6x + 5$ is decreasing. [3 marks]

Ans: $f'(x) = 2x - 6$. The function decreases where $f'(x) < 0$: $2x - 6 < 0 \Rightarrow x < 3$. So f is decreasing on $(-\infty, 3)$. (It increases for $x > 3$, with the turning point at x = 3.)

Q8. Evaluate $\lim_{x\to1} (x^3 - 1)/(x - 1)$. [3 marks]

Ans: Substitution gives 0/0. Factor $x^3 - 1 = (x - 1)(x^2 + x + 1)$; cancel (x – 1): expression = $x^2 + x + 1$. Then $\lim_{x\to1}(x^2 + x + 1) = 1 + 1 + 1 = 3$.

Section C — Long-answer, multi-step (5 × 2 = 10 marks)

Q9. Find the local maximum and local minimum values of $f(x) = x^3 - 3x^2 + 4$ using the second-derivative test. State clearly which point is which. [5 marks]

Ans: $f'(x) = 3x^2 - 6x = 3x(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$. $f''(x) = 6x - 6$. At x = 0: $f''(0) = -6 < 0 \Rightarrow$ maximum; f(0) = 4. At x = 2: $f''(2) = 6 > 0 \Rightarrow$ minimum; $f(2) = 8 - 12 + 4 = 0$. Hence local maximum value 4 at x = 0 and local minimum value 0 at x = 2.

Q10. A firm's total revenue is $R(x) = 20x - 0.5x^2$ and total cost is $C(x) = 4x + 40$, where x is the number of units. (a) Find the marginal revenue and marginal cost. (b) Find the output x at which profit is maximised (MR = MC) and the maximum profit. [5 marks]

Ans: (a) $MR = dR/dx = 20 - x$; $MC = dC/dx = 4$. (b) Profit is maximised where MR = MC: $20 - x = 4 \Rightarrow x = 16$. Profit $P(x) = R - C = (20x - 0.5x^2) - (4x + 40) = 16x - 0.5x^2 - 40$; $P'(x) = 16 - x = 0 \Rightarrow x = 16$, and $P''(x) = -1 < 0$ confirms a maximum. Maximum profit = $P(16) = 16\cdot16 - 0.5\cdot256 - 40 = 256 - 128 - 40 = 88$. So profit is maximised at x = 16 units with maximum profit Rs 88.

★ TOPPER TEMPLATE — 3 marks: 'Evaluate $\lim_{x\to 2} (x^2-4)/(x-2)$. Justify each step.'

Annual + SQP

Step 1 [1 mark]	**Test direct substitution and identify the form**	Substituting x = 2 gives $(2^2-4)/(2-2) = 0/0$, which is an INDETERMINATE form. Direct substitution fails, so the expression must be simplified before the limit can be taken. Naming the 0/0 form explicitly earns this mark.
Step 2 [1 mark]	**Factorise and cancel the common factor**	Factor the numerator: $x^2-4 = (x-2)(x+2)$. So $(x^2-4)/(x-2) = [(x-2)(x+2)]/(x-2) = (x+2)$, cancelling the common (x–2) factor (valid because $x \to 2$ means $x \neq 2$, so the factor is non-zero).
Step 3 [1 mark]	**Take the limit of the simplified expression**	Now $\lim_{x\to 2}(x+2) = 2 + 2 = 4$. Therefore $\lim_{x\to 2}(x^2-4)/(x-2) = 4$. The limit exists and equals 4 even though the original function is undefined at x = 2.

COMMON LOSS OF MARKS:

- Writing '0/0 = 0' or '0/0 = does not exist' instead of recognising it as indeterminate and simplifying.
- Cancelling without stating that $x \neq 2$ justifies removing the (x–2) factor.
- Forgetting to substitute at the end, leaving the answer as '(x+2)' instead of the number 4.

★ TOPPER TEMPLATE — 4 marks: 'The total cost of producing x units is $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost and evaluate it at x = 50.'

Annual

Step 1 [1 mark]	**State what marginal cost means**	Marginal cost is the rate of change of total cost with respect to output, i.e. MC = dC/dx. It approximates the cost of producing ONE additional unit. So we differentiate C(x).
Step 2 [1 mark]	**Differentiate the cost function**	$MC = dC/dx = d/dx(0.005x^3 - 0.02x^2 + 30x + 5000) = 0.005 \cdot 3x^2 - 0.02 \cdot 2x + 30 = 0.015x^2 - 0.04x + 30$ (the constant 5000 differentiates to 0).
Step 3 [1 mark]	**Evaluate at the required output**	At x = 50: $MC = 0.015 \cdot (50)^2 - 0.04 \cdot (50) + 30 = 0.015 \cdot 2500 - 2 + 30 = 37.5 - 2 + 30 = 65.5$.
Step 4 [1 mark]	**Interpret the result in context**	The marginal cost at x = 50 is Rs 65.5, meaning that when 50 units are already being produced, producing the 51st unit costs approximately Rs 65.5. Always close an applied-calculus answer with an interpretation in the units of the problem.

COMMON LOSS OF MARKS:

- Differentiating the constant 5000 to anything other than 0.
- Arithmetic slip on 0.015·2500 — write 2500, not 250 ($50^2 = 2500$).
- Giving the number 65.5 without the interpretation sentence; the marking scheme reserves a mark for the economic meaning.

★ TOPPER TEMPLATE — 5 marks: 'Find the maximum or minimum value of $f(x) = x^3 - 3x^2 + 4$, using the second-derivative test.'

Most years

Step 1 [1 mark]	**Differentiate and set f'(x) = 0**	$f'(x) = 3x^2 - 6x$. For stationary points set $f'(x) = 0$: $3x^2 - 6x = 0 \Rightarrow 3x(x - 2) = 0 \Rightarrow x = 0$ or x = 2.
Step 2 [1 mark]	**Find the second derivative**	$f''(x) = d/dx(3x^2 - 6x) = 6x - 6$.
Step 3 [1 mark]	**Apply the second-derivative test at each point**	At x = 0: $f''(0) = 6 \cdot 0 - 6 = -6 < 0 \Rightarrow$ MAXIMUM. At x = 2: $f''(2) = 6 \cdot 2 - 6 = 6 > 0 \Rightarrow$ MINIMUM. (Recall: f'' negative → maximum, f'' positive → minimum.)
Step 4 [1 mark]	**Compute the extreme values**	Maximum value = $f(0) = 0^3 - 3 \cdot 0^2 + 4 = 4$ (at x = 0). Minimum value = $f(2) = 2^3 - 3 \cdot 2^2 + 4 = 8 - 12 + 4 = 0$ (at x = 2).
Step 5 [1 mark]	**State the conclusion clearly**	Hence f has a LOCAL MAXIMUM of 4 at x = 0 and a LOCAL MINIMUM of 0 at x = 2. Stating which point is which, with values, secures the final mark.

COMMON LOSS OF MARKS:

- Reversing the second-derivative test (f'' > 0 wrongly called a maximum).
- Computing f(0) or f(2) carelessly — show 8 – 12 + 4 = 0 explicitly.
- Stopping at the x-values without giving the actual maximum/minimum VALUES of f.

MARKING SCHEME — GENERAL NOTES

- Limits: always test substitution first; on 0/0 show the factorisation and cancellation before re-substituting — the simplification carries the method mark.
- Differentiation: NAME the rule used (power/product/quotient/chain). For the chain rule the inner derivative must appear explicitly or a mark is lost.
- First principles: the difference-quotient limit $[f(x+h)-f(x)]/h$ must be written; cancel h before letting $h \to 0$.

- Marginal analysis: MC = dC/dx and MR = dR/dx; finish with a one-line interpretation in rupees per unit — a mark is reserved for the economic meaning.
- Maxima-minima: solve $f'(x) = 0$, then apply the second-derivative test ($f'' < 0 \Rightarrow$ maximum, $f'' > 0 \Rightarrow$ minimum), and report the actual extreme VALUE of f, not only the x-coordinate.