CHAPTER 4

Calculus

CBSE Class 11 · Applied Mathematics (241) · Unit 4

CBSE · Applied Mathematics · Class 11

WHAT THIS CHAPTER DOES

A State the domain and range of a function, and recognise the standard function types.

B Evaluate a limit — including the 0/0 form — by factorising before substituting.

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's mission

1. State the domain and range of a function, and recognise the standard function types.
2. Evaluate a limit — including the 0/0 form — by factorising before substituting.
3. Differentiate any polynomial, product, quotient or composite function using the right rule.
4. Find marginal cost/revenue and the maximum or minimum of a function, and interpret the result.

WHY THIS MATTERS

Why this chapter matters

1. Calculus is the heaviest unit of Class 11 Applied Maths — roughly 18-20 of the 80 external marks.
2. It is the engine behind business mathematics: marginal cost, marginal revenue and profit-maximisation all come from derivatives.
3. Everything here is the foundation for Class 12 — integration, differential equations and further optimisation all stand on these rules.

TOPIC

A

Functions — domain and range

TOPIC

What a function is, and its domain and range

THE IDEA OF A FUNCTION

A function is a rule that assigns to each input x exactly ONE output y, written y = f(x). The key word is 'exactly one' — every permitted input gets a single, definite output, which is why $y = x^2$ is a function but a rule giving two outputs for one input is not. In Applied Maths the input is often a business

DOMAIN — THE ALLOWED INPUTS

The domain is the set of all x-values for which the function gives a real output. Most functions accept every real number, but three situations restrict it: a DENOMINATOR may not be zero (so $1/(x-3)$ excludes x = 3); the expression under a SQUARE ROOT must be

RANGE — THE RESULTING OUTPUTS

The range is the set of all output values f(x) actually takes as x runs over the domain. For $y = x^2$, no matter what real x you choose the output is never negative, so the range is $y \geqslant 0$ even though the domain is all real numbers. For $y = e^x$ the output is always positive (range $y > 0$),

WHY FUNCTIONS OPEN THE CALCULUS

Calculus studies how the OUTPUT of a function changes as its INPUT changes, so every later idea — limit, derivative, marginal cost, maximum — is a statement about a function. That is why the chapter opens here: you cannot ask 'how fast is cost rising?' until 'cost as a function of output' is

TOPIC

B

Limits and continuity

THEOREM · LOAD-BEARING RESULT

The limit of a function — and the 0/0 form

> $\lim_{x \to a} f(x) = L$ means f(x) gets arbitrarily close to L as x approaches a from both sides. The left-hand limit ($x \to a^-$) and right-hand limit ($x \to a^+$) must agree for the limit to exist. The limit equals f(a) only when f is continuous at a.

STATEMENT

Procedure: first try direct substitution. If f(a) is defined, that value is the limit. If substitution gives the INDETERMINATE form 0/0, the function must be simplified — typically by

WHY THIS MATTERS

- Limits make the derivative possible: the derivative is itself a limit (of a difference quotient as $h \to 0$)
- Mastering the 0/0 factorisation routine here is what makes first-principles differentiation and every later 'rate of change' idea work, and limit questions are reliable 2-3 mark earners in their own right.

WATCH OUT FOR

NOTE 0/0 is INDETERMINATE — it does NOT mean the limit is 0 or that it fails to exist. You must remove the common factor and re-substitute. Also, the limit of f as $x \to a$ can exist even when f(a) itself is undefined — the limit is about approaching a, not arriving at it.

WORKED EXAMPLE

Evaluate two limits, one of them a 0/0 form

1. QUESTION: (a) $\lim_{x \to 3} (2x^2 - 5)$. (b) $\lim_{x \to 3} (x^2 - 9)/(x - 3)$.
2. (a) f is a polynomial, so it is continuous everywhere — substitute directly: $2 \cdot (3)^2 - 5 = 2 \cdot 9 - 5 = 18 - 5 = 13$. So the limit is 13.
3. (b) Substituting $x = 3$ gives $(9 - 9)/(3 - 3) = 0/0$ — an INDETERMINATE form, so simplify first.
4. Factor the numerator: $x^2 - 9 = (x - 3)(x + 3)$. Cancel the common $(x - 3)$: $(x^2 - 9)/(x - 3) = (x + 3)$ for $x \neq 3$.
5. Now substitute: $\lim_{x \to 3}(x + 3) = 3 + 3 = 6$. So the limit is 6.
6. TAKEAWAY: always try substitution first; if you hit 0/0, factorise, cancel, and substitute again — never report 0/0 as the answer.

TOPIC

When is a function continuous at a point?

THE THREE-PART TEST

A function f is continuous at x = a when three conditions ALL hold: (i) f(a) exists (the function is defined at a), (ii) lim(x→a) f(x) exists (the left-hand and right-hand limits agree), and (iii) those two are equal, lim(x→a) f(x) = f(a). Intuitively, continuity means you can draw the graph through x = a

LEFT-HAND AND RIGHT-HAND LIMITS

The left-hand limit (LHL) is the value f(x) approaches as x → a from values smaller than a (written $x \to a^-$); the right-hand limit (RHL) is the approach from values larger than a ($x \to a^+$). For an ordinary limit to exist, LHL must equal RHL. Where a function is defined by different

CONTINUITY IN PLAIN PICTURES

Polynomials (like $x^2 - 3x + 2$) are continuous everywhere — no input ever breaks them. A rational function such as 1/(x–2) is continuous everywhere EXCEPT where the denominator is zero (here x = 2), which produces a break. A piecewise function can have a jump where the

WHY CONTINUITY MATTERS FOR CALCULUS

Continuity is the gateway to differentiability: a function must be continuous at a point to have a derivative there (though continuity alone is not enough). It also justifies the shortcut you use constantly — for a continuous function the limit IS just the value, so direct substitution works.

TOPIC

C

Differentiation — first principles and rules

THEOREM · LOAD-BEARING RESULT

The derivative — definition and the standard rules

> The derivative of f at x is $f'(x) = \lim(h \to 0)\ [f(x+h) - f(x)] / h$, when this limit exists. Geometrically it is the slope of the tangent to $y = f(x)$; physically it is the instantaneous rate of change of f. The standard rules let you avoid the limit each time.

STATEMENT

Rules: POWER $d/dx(x^n) = n \cdot x^{n-1}$; CONSTANT $d/dx(c) = 0$; CONSTANT MULTIPLE $d/dx(c \cdot f) = c \cdot f'$; SUM/DIFFERENCE $(u \pm v)' = u' \pm v'$; PRODUCT $(uv)' = u'v + uv'$; QUOTIENT $(u/v)' = (u'v - uv')/$

WHY THIS MATTERS

- Differentiation is the most-tested skill in the chapter (~24% of its marks) and the tool behind every application — rate of change, marginal cost, increasing/decreasing and maxima-minima all reduce to 'find a derivative'
- First principles is examined occasionally to show you

WATCH OUT FOR

NOTE The product rule is $(uv)' = u'v + uv'$, NOT $u' \cdot v'$. The quotient rule numerator is $(u'v - uv')$, in that order — sign and order matter. In the chain rule you MUST multiply by the derivative of the inner function; dropping it is the top differentiation error.

WORKED EXAMPLE

Differentiate from first principles, then by the power rule

1. QUESTION: Differentiate $f(x) = x^2$ from first principles, and confirm with the power rule.

2. FIRST PRINCIPLES: $f'(x) = \lim(h\to 0)\ [f(x+h) - f(x)]/h = \lim(h\to 0)\ [(x+h)^2 - x^2]/h$.

3. Expand the numerator: $(x+h)^2 - x^2 = x^2 + 2xh + h^2 - x^2 = 2xh + h^2$. So the quotient $= (2xh + h^2)/h = 2x + h$ (cancel h, valid as $h \neq 0$ in the limit).

4. Take the limit: $\lim(h\to 0)(2x + h) = 2x$. So $f'(x) = 2x$.

5. POWER-RULE CHECK: $d/dx(x^2) = 2\cdot x^{2-1} = 2x^1 = 2x$. ✓ Same answer.

6. TAKEAWAY: first principles uses the difference-quotient limit directly; the power rule is the shortcut it proves. Always cancel the h before letting $h \to 0$.

WORKED EXAMPLE

Apply the product, quotient and chain rules

1. QUESTION: Differentiate (a) $y = (2x + 1)(x^2 - 3)$; (b) $y = (3x^2)/(x + 1)$; (c) $y = (2x + 1)^5$.

2. (a) PRODUCT RULE with $u = 2x+1$ ($u' = 2$), $v = x^2-3$ ($v' = 2x$): $y' = u'v + uv' = 2\cdot(x^2-3) + (2x+1)\cdot 2x = 2x^2 - 6 + 4x^2 + 2x = 6x^2 + 2x - 6$.

3. (b) QUOTIENT RULE with $u = 3x^2$ ($u' = 6x$), $v = x+1$ ($v' = 1$): $y' = (u'v - uv')/v^2 = (6x\cdot(x+1) - 3x^2\cdot 1)/(x+1)^2 = (6x^2 + 6x - 3x^2)/(x+1)^2 = (3x^2 + 6x)/(x+1)^2$.

4. (c) CHAIN RULE: outer is $(\cdot)^5$, inner is $(2x+1)$ with inner derivative 2. $y' = 5(2x+1)^4 \cdot 2 = 10(2x+1)^4$.

5. TAKEAWAY: match the structure to the rule — product → $u'v+uv'$, quotient → $(u'v-uv')/v^2$, composite → outer'×inner'. In (c) the factor of 2 from the inner derivative is exactly what most students forget.

TOPIC

D

Applications — rate of change, marginal cost & revenue

THEOREM · LOAD-BEARING RESULT

The derivative as a rate of change — marginal cost and revenue

> The derivative dy/dx measures the instantaneous rate of change of y with respect to x. In economics, if C(x) is the total cost of producing x units then the MARGINAL COST is MC = dC/dx; if R(x) is total revenue then the MARGINAL REVENUE is MR = dR/dx. Each approximates the effect of producing/selling one more unit.

STATEMENT

MC = dC/dx = C'(x) approximates the cost of the (x+1)th unit. MR = dR/dx = R'(x) approximates the revenue from selling the (x+1)th unit. Profit P(x) = R(x) – C(x), and dP/dx = MR – MC;

WHY THIS MATTERS

- This is the bridge between calculus and commerce, and it is heavily examined in Applied Maths (~25% of the chapter's marks together with maxima-minima)
- The marginal concept — the change caused by one more unit — is the single most important economic idea the derivative delivers, and

WATCH OUT FOR

NOTE Marginal cost is the DERIVATIVE of total cost, not the average cost (C(x)/x) — the two are different. Always finish a marginal-quantity answer with an interpretation in the problem's units (rupees per unit); marking schemes reserve a mark for the economic meaning, not just the number.

WORKED EXAMPLE

Find and interpret marginal cost and marginal revenue

1. QUESTION: A firm has total cost $C(x) = 0.01x^2 + 5x + 200$ and total revenue $R(x) = 12x$. (a) Find MC at $x = 50$. (b) Find MR. (c) At $x = 50$, is profit rising or falling?

2. (a) $MC = dC/dx = 0.01 \cdot 2x + 5 = 0.02x + 5$. At $x = 50$: $MC = 0.02 \cdot 50 + 5 = 1 + 5 = 6$. So the 51st unit costs about Rs 6.

3. (b) $MR = dR/dx = 12$ (revenue is linear, so each extra unit brings a constant Rs 12).

4. (c) $dP/dx = MR - MC = 12 - 6 = 6 > 0$ at $x = 50$, so PROFIT IS RISING — producing more still adds to profit at this output.

5. INTERPRETATION: since MR (12) exceeds MC (6), each additional unit earns more than it costs, so the firm should expand output beyond 50 toward where MR = MC.

6. TAKEAWAY: $MC = dC/dx$, $MR = dR/dx$; compare them — while MR > MC profit grows; the optimum is where MR = MC.

TOPIC

E

Applications — increasing/decreasing and maxima-minima

THEOREM · LOAD-BEARING RESULT

Increasing/decreasing functions and the maxima-minima tests

> A function is INCREASING on an interval where $f'(x) > 0$ and DECREASING where $f'(x) < 0$. Points where $f'(x) = 0$ are STATIONARY points — candidates for a maximum or minimum. The second-derivative test classifies them.

STATEMENT

Procedure: (1) compute $f'(x)$ and solve $f'(x) = 0$ to find stationary points; (2) compute $f''(x)$; (3) at each stationary point, if $f''(x) < 0$ the point is a LOCAL MAXIMUM (curve concave down), if $f''(x) > 0$ it is

WHY THIS MATTERS

- Optimisation is the climax of the chapter — maximising profit, minimising cost or material, finding the most efficient output
- Together with marginal analysis it carries about a quarter of the chapter's marks, and the 5-mark optimisation question is where strong scripts separate from

WATCH OUT FOR

NOTE The second-derivative test is the OPPOSITE of the naive guess: $f''(x) < 0$ gives a MAXIMUM and $f''(x) > 0$ gives a MINIMUM. Also, stationary points are where $f'(x) = 0$, NOT where $f'(x)$ is largest. And always report the actual maximum/minimum value $f(x)$, not merely the x-coordinate.

WORKED EXAMPLE

Find the maximum and minimum of a cubic

1. QUESTION: Find the local maximum and minimum values of $f(x) = x^3 - 3x^2 + 4$.
2. STEP 1 — $f'(x) = 3x^2 - 6x$. Set $f'(x) = 0$: $3x(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$ (the stationary points).
3. STEP 2 — $f''(x) = 6x - 6$.
4. STEP 3 — At $x = 0$: $f''(0) = -6 < 0 \Rightarrow$ MAXIMUM. At $x = 2$: $f''(2) = 6 > 0 \Rightarrow$ MINIMUM.
5. STEP 4 — Values: $f(0) = 0 - 0 + 4 = 4$ (local maximum); $f(2) = 8 - 12 + 4 = 0$ (local minimum).
6. CONCLUSION: local maximum value 4 at $x = 0$, local minimum value 0 at $x = 2$. TAKEAWAY: $f'=0$ finds the candidates, f'' classifies them (negative $\Rightarrow$ max), and you must compute the actual values.

TOPIC

Limit of a function vs value of the function

TRAP → TRUTH

× **MISTAKE** lim(x→a) f(x) is just f(a) — plug in a and you are done.

✓ **CORRECT** The limit describes what f(x) APPROACHES as x gets close to a, which need not equal f(a) — and f(a) may not even exist. For lim(x→2) $(x^2–4)/(x–2)$, direct substitution gives 0/0 (undefined), yet the limit is 4 after cancelling (x–2). Substitution works ONLY when the function is continuous at a; when it gives 0/0 you must factorise or simplify first.

TOPIC

The 0/0 indeterminate form

TRAP → TRUTH

× **MISTAKE** If substitution gives 0/0, the limit does not exist (or equals 0, or equals 1).

✓ **CORRECT** 0/0 is an INDETERMINATE form — it tells you the answer is hidden, not that it is zero or undefined. You must remove the common factor causing both zeros (by factorising and cancelling) and THEN substitute. $\lim_{x\to 3}(x^2-9)/(x-3) = \lim (x+3) = 6$, not 0/0.

TOPIC

Power rule on roots and reciprocals

TRAP → TRUTH

× **MISTAKE** The derivative of $\sqrt{x}$ is $\sqrt{x}$, and the derivative of $1/x$ is $1/x$ (or you cannot differentiate them at all).

✓ **CORRECT** Rewrite as a power FIRST, then apply $d/dx(x^n) = n \cdot x^{n-1}$. $\sqrt{x} = x^{(1/2)}$ ⇒ derivative = $(1/2)x^{(-1/2)} = 1/(2\sqrt{x})$. And $1/x = x^{-1}$ ⇒ derivative = $-1 \cdot x^{-2} = -1/x^2$. The power rule handles fractional and negative exponents — you just have to write the function as a power before differentiating.

TOPIC

Product rule

TRAP → TRUTH

× **MISTAKE** The derivative of a product is the product of the derivatives: $(uv)' = u' \cdot v'$.

✓ **CORRECT** This is FALSE. The product rule is $(uv)' = u'v + uv'$ — differentiate the first times the second PLUS the first times the derivative of the second. Test it on $y = x \cdot x = x^2$: the true derivative is $2x$, but $u' \cdot v'$ would give $1 \cdot 1 = 1$ (wrong). Two terms, never one.

TOPIC

Chain rule — forgetting the inner derivative

TRAP → TRUTH

× **MISTAKE** $d/dx[(2x+1)^5] = 5(2x+1)^4$.

✓ **CORRECT** You must multiply by the derivative of the INSIDE function. $d/dx[(2x+1)^5] = 5(2x+1)^4 \cdot d/dx(2x+1) = 5(2x+1)^4 \cdot 2 = 10(2x+1)^4$. The chain rule is 'derivative of the outer (keeping the inner intact) × derivative of the inner'. Dropping the inner derivative is the top chain-rule error.

TOPIC

Sign of the derivative and increasing/decreasing

TRAP → TRUTH

× **MISTAKE** If $f'(x)$ is positive the function is decreasing; a maximum is where $f'(x)$ is largest.

✓ **CORRECT** $f'(x) > 0$ means f is INCREASING (slope upward); $f'(x) < 0$ means DECREASING. A maximum or minimum occurs where $f'(x) = 0$ (a stationary point), NOT where f' is largest. At a maximum the slope is zero and changes from + to –; at a minimum from – to +. Confusing the sign of the derivative with the size of the function is a common reasoning slip.

TOPIC

Second-derivative test for maxima vs minima

TRAP → TRUTH

× **MISTAKE** If $f''(x) > 0$ at a stationary point, it is a maximum.

✓ **CORRECT** It is the OPPOSITE. At a stationary point ($f'(x) = 0$): if $f''(x) < 0$ the point is a MAXIMUM (curve bends down, like a hill-top); if $f''(x) > 0$ it is a MINIMUM (curve bends up, like a valley). Mnemonic: 'second derivative negative → maximum' (both have the 'down' feel). Swapping these costs the final mark in every maxima-minima question.

TOPPER TEMPLATE · MARK-BY-MARK

3 marks: 'Evaluate $\lim(x\to2)\ (x^2-4)/(x-2)$. Justify each step.'

1 — TEST DIRECT SUBSTITUTION AND IDENTIFY THE FORM (1 m)

Substituting $x = 2$ gives $(2^2-4)/(2-2) = 0/0$, which is an INDETERMINATE form. Direct substitution fails, so the expression must be simplified before the limit can be taken. Naming the 0/0 form explicitly earns this mark.

2 — FACTORISE AND CANCEL THE COMMON FACTOR (1 m)

Factor the numerator: $x^2-4 = (x-2)(x+2)$. So $(x^2-4)/(x-2) = [(x-2)(x+2)]/(x-2) = (x+2)$, cancelling the common $(x-2)$ factor (valid because $x \to 2$ means $x \neq 2$, so the factor is non-zero).

3 — TAKE THE LIMIT OF THE SIMPLIFIED EXPRESSION (1 m)

Now $\lim(x\to2)(x+2) = 2 + 2 = 4$. Therefore $\lim(x\to2)(x^2-4)/(x-2) = 4$. The limit exists and equals 4 even though the original function is undefined at $x = 2$.

TOPPER TEMPLATE · MARK-BY-MARK

4 marks: 'The total cost of producing x units is $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the

1 STATE WHAT MARGINAL COST MEANS (1 m)

Marginal cost is the rate of change of total cost with respect to output, i.e. MC = dC/dx. It approximates the cost of producing ONE additional unit. So we differentiate C(x).

2 DIFFERENTIATE THE COST FUNCTION (1 m)

$MC = dC/dx = d/dx(0.005x^3 - 0.02x^2 + 30x + 5000) = 0.005 \cdot 3x^2 - 0.02 \cdot 2x + 30 = 0.015x^2 - 0.04x + 30$ (the constant 5000 differentiates to 0).

3 EVALUATE AT THE REQUIRED OUTPUT (1 m)

At x = 50: $MC = 0.015 \cdot (50)^2 - 0.04 \cdot (50) + 30 = 0.015 \cdot 2500 - 2 + 30 = 37.5 - 2 + 30 = 65.5$.

4 INTERPRET THE RESULT IN CONTEXT (1 m)

The marginal cost at x = 50 is Rs 65.5, meaning that when 50 units are already being produced, producing the 51st unit costs approximately Rs 65.5. Always close an applied-calculus answer with an interpretation in the units of the problem.

TOPPER TEMPLATE · MARK-BY-MARK

5 marks: 'Find the maximum or minimum value of $f(x) = x^3 - 3x^2 + 4$, using the second-

1	**DIFFERENTIATE AND SET F'(X) = 0** 1 m	$f'(x) = 3x^2 - 6x$. For stationary points set $f'(x) = 0$: $3x^2 - 6x = 0 \Rightarrow 3x(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$.
2	**FIND THE SECOND DERIVATIVE** 1 m	$f''(x) = d/dx(3x^2 - 6x) = 6x - 6$.
3	**APPLY THE SECOND-DERIVATIVE TEST AT EACH POINT** 1 m	At $x = 0$: $f''(0) = 6\cdot 0 - 6 = -6 < 0 \Rightarrow$ MAXIMUM. At $x = 2$: $f''(2) = 6\cdot 2 - 6 = 6 > 0 \Rightarrow$ MINIMUM. (Recall: f'' negative → maximum, f'' positive → minimum.)
4	**COMPUTE THE EXTREME VALUES** 1 m	Maximum value = $f(0) = 0^3 - 3\cdot 0^2 + 4 = 4$ (at $x = 0$). Minimum value = $f(2) = 2^3 - 3\cdot 2^2 + 4 = 8 - 12 + 4 = 0$ (at $x = 2$).
5	**STATE THE CONCLUSION CLEARLY** 1 m	Hence f has a LOCAL MAXIMUM of 4 at $x = 0$ and a LOCAL MINIMUM of 0 at $x = 2$. Stating which point is which, with values, secures the final mark.

PYQ PATTERNS

Top PYQ patterns to drill

#1	Differentiate a function using the standard rules (power, product, quotient or chain)	e.g. find dy/dx for y = (2x+1)(x^2-3) or y = ($3x^2$)/(x+1). (3 marks) — Almost every annual paper + SQP
#2	Evaluate a 0/0 limit by factorisation	e.g. lim(x→2) (x^2-4)/(x−2). (2-3 marks) — SQP + annual pattern
#3	Find the marginal cost or marginal revenue from a cost/revenue function and interpret it	e.g. given C(x), find MC at x = 10. (3 marks) — Annual
#4	Find the maximum/minimum value of a function using the first- or second-derivative test	e.g. maximise profit P(x) = R(x) − C(x). (5

RECAP · MEMORISE THESE

Recap — the calculus toolkit

1 Functions — f(x) maps each input to one output. Domain excludes division-by-zero, negative roots and non-positive log arguments; range is the set of outputs.

2 Limits & continuity — Substitute first; if 0/0, factorise, cancel, re-substitute. Continuous at a means f(a) exists, the limit exists, and they are equal.

3 Differentiation — $f'(x) = \lim(h\to 0)[f(x+h) - f(x)]/h$. Power: $n \cdot x^{n-1}$. Product: $u'v+uv'$. Quotient: $(u'v-uv')/v^2$. Chain: outer'×inner'. $d/dx(e^x)=e^x$, $d/dx(\ln x)=1/x$.

4 Rate of change & marginal — Derivative = rate of change. MC

5 Increasing/decreasing & extrema — $f' > 0$ increasing, $f' < 0$

WHAT'S NEXT

What's next

- → Unit 5 — Probability, and the Financial Mathematics applications that reuse these derivatives.
- → Sit the 15-MCQ Quick Drill for this chapter — under 25 minutes, target ⩾ 12/15.
- → Then the full school-pattern paper — 30 marks, internal-exam style, with full model answers.

You've mastered Unit 4 — Calculus.

Functions · limits · differentiation · marginal analysis · maxima-minima — now prove it on the paper.

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