ANSWER KEY & MARKING SCHEME · CBSE CLASS 11

Relations and Functions

Mathematics · Chapter 2 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Very Short Answer

Q1. If A = {1, 2} and B = {3, 4}, write the number of elements in A × B. [1 mark]

Ans: $|A \times B| = |A| \cdot |B| = 2 \cdot 2 = 4$.

Q2. Find x if (x + 2, 5) = (4, 5). [1 mark]

Ans: First coordinates: $x + 2 = 4 \Rightarrow x = 2$.

Q3. State the range of the constant function f(x) = 7. [1 mark]

Ans: Range = {7} (the only output is 7).

Q4. Write the range of the signum function on R. [1 mark]

Ans: Range = {−1, 0, 1}.

Q5. If f(x) = 3x and g(x) = x + 1, find (f + g)(2). [1 mark]

Ans: $(f + g)(2) = f(2) + g(2) = 6 + 3 = 9$.

Section B — Short Answer

Q6. If A = {1, 2, 3}, write the relation R = {(x, y) : y = x + 1, x ∈ A, y ∈ A} in roster form. [2 marks]

Ans: $x = 1 \Rightarrow y = 2 \in A$ ✓ (1, 2). $x = 2 \Rightarrow y = 3 \in A$ ✓ (2, 3). $x = 3 \Rightarrow y = 4 \notin A$ ×. Hence R = {(1, 2), (2, 3)}.

Q7. If f(x) = 2x + 1 and g(x) = x², find (f∘g)(2) and (g∘f)(2). Are they equal? [2 marks]

Ans: $(f\circ g)(2) = f(g(2)) = f(4) = 2\cdot4 + 1 = 9$. $(g\circ f)(2) = g(f(2)) = g(5) = 25$. Not equal. Composition is non-commutative.

Q8. Find the domain of f(x) = 1/(x − 5). [2 marks]

Ans: Need denominator ≠ 0: $x - 5 \neq 0 \Rightarrow x \neq 5$. Domain = $R \setminus \{5\}$.

Q9. If A = {a, b}, write all subsets of A × A that are reflexive relations. [2 marks]

Ans: A × A = {(a,a),(a,b),(b,a),(b,b)}. A reflexive relation must include (a,a) and (b,b). Optional pairs are (a,b), (b,a) — independently. So $2^2 = 4$ reflexive relations: {(a,a),(b,b)}; {(a,a),(b,b),(a,b)}; {(a,a),(b,b),(b,a)}; {(a,a),(b,b),(a,b),(b,a)}.

Section C — Long Answer

Q10. Find the domain and range of f(x) = √(9 − x²). [3 marks]

Ans: Domain: need $9 - x^2 \geq 0 \Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3$, so domain = [−3, 3]. Range: $f(x) \geq 0$ (square root). Maximum at x = 0: $f(0) = \sqrt{9} = 3$. Minimum at $x = \pm3$: f = 0. Range = [0, 3].

Q11. Show that f: N → N defined by f(x) = 2x is one-one but not onto. [3 marks]

Ans: One-one: let $f(a) = f(b) \Rightarrow 2a = 2b \Rightarrow a = b$. ✓. Onto: take $y = 1 \in$ codomain N. Then $2x = 1 \Rightarrow x = 1/2 \notin$ N. So 1 has no pre-image. Hence f is one-one but not onto.

Q12. If f(x) = x² + 1 and g(x) = 2x − 3, find (f∘g)(x) and (g∘f)(x). [3 marks]

Ans: $(f\circ g)(x) = f(g(x)) = f(2x - 3) = (2x - 3)^2 + 1 = 4x^2 - 12x + 9 + 1 = 4x^2 - 12x + 10$. $(g\circ f)(x) = g(f(x)) = g(x^2 + 1) = 2(x^2 + 1) - 3 = 2x^2 + 2 - 3 = 2x^2 - 1$.

Section D — Application

Q13. Draw the graph of the greatest integer function $f(x) = [x]$ for $-2 \leqslant x < 3$ and find its range on this interval. **[4 marks]**

> **Ans:** On $x \in [-2,-1)$ $f = -2$; $[-1,0)$ $f = -1$; $[0,1)$ $f = 0$; $[1,2)$ $f = 1$; $[2,3)$ $f = 2$. Sketch: five horizontal segments at $y = -2, -1, 0, 1, 2$. Filled dot at the LEFT end of each step, open circle at the right end. Range on $[-2, 3) = \{-2, -1, 0, 1, 2\}$.

Q14. Let $f(x) = (x + 1)/(x - 1)$, $x \neq 1$. Find (a) the domain, (b) $f(2)$, $f(0)$, $f(-1)$, (c) check whether f is one-one on its domain. **[4 marks]**

> **Ans:** (a) Need $x \neq 1$, so domain = $R \setminus \{1\}$. (b) $f(2) = 3/1 = 3$; $f(0) = 1/(-1) = -1$; $f(-1) = 0/(-2) = 0$. (c) Let $f(a) = f(b)$: $(a+1)/(a-1) = (b+1)/(b-1) \Rightarrow (a+1)(b-1) = (a-1)(b+1) \Rightarrow ab - a + b - 1 = ab + a - b - 1 \Rightarrow -a + b = a - b \Rightarrow 2b = 2a \Rightarrow a = b$. Hence f is one-one on its domain.

★ TOPPER ANSWER TEMPLATES

5 TEMPLATES · MEMORISE THE FORMAT

★ TOPPER TEMPLATE — Find A×B or list a relation in roster form (2 marks)

Asked in roughly 9 out of every 10 CBSE papers

Step Write what A and B are; state |A| and |B| [0.5 mark]	**Setup**	Given $A = \{1, 2, 3\}$, $B = \{a, b\}$. $\|A\| = 3$, $\|B\| = 2$, so $\|A \times B\| = 6$.
Step List pairs row-major: fix first coordinate, sweep second [1.0 mark]	**List**	$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$.
Step If a relation rule is given, filter and write the final R [0.5 mark]	**Filter + state**	If $R = \{(x,y) : y \text{ is a vowel}\}$, both a and b are listed but only the ones with $y \in \{a\}$ qualify. Hence $R = \{(1,a), (2,a), (3,a)\}$.

COMMON LOSS OF MARKS:

- Missing pairs because student jumps coordinates randomly instead of row-major.
- Writing pairs as unordered {1, a} instead of ordered (1, a).
- Not re-checking that the listed elements lie in the right set when a rule is applied.

★ TOPPER TEMPLATE — Find domain and range of a real function (3 marks)

8 out of every 10 papers; usually √, 1/(x−a), or |x|

Step State the formula and write the 'allowed-input' condition [0.5 mark]	**Condition**	$f(x) = \sqrt{9 - x^2}$. For √ to be defined we need $9 - x^2 \geq 0$.
Step Solve the inequality cleanly [1.0 mark]	**Solve**	$9 - x^2 \geq 0 \Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3$.
Step State domain in interval notation [0.5 mark]	**Domain**	Domain = $[-3, 3]$.
Step Find range using extremes and the nature of the function [1.0 mark]	**Range**	Since $\sqrt{(\cdot)} \geq 0$ always, min value is 0 (at $x = \pm 3$). Max value at $x = 0 \Rightarrow f(0) = 3$. Range = $[0, 3]$.

COMMON LOSS OF MARKS:

- Writing the domain as 'all real numbers' without checking the √ or denominator condition.
- Reporting range as R instead of [0, 3] because student didn't note √ ⩾ 0.
- Using { } set-builder where the marking scheme wants interval notation (both are usually accepted, but mixing them muddles answers).

★ TOPPER TEMPLATE — Show f is one-one / onto / bijective (3 marks)

7 out of every 10 papers

Step State definitions you'll use [0.5 mark]	**Definitions**	f is one-one if $f(a) = f(b) \Rightarrow a = b$. f is onto if every y in codomain has some x with $f(x) = y$.
Step Prove (or disprove) one-one [1.0 mark]	**One-one**	Let $f(a) = f(b)$. Then $2a = 2b \Rightarrow a = b$. Hence f is one-one.
Step Prove (or disprove) onto with a clean counter-example or general proof [1.0 mark]	**Onto**	Take $y = 1 \in N$ (codomain). We need $x \in N$ with $2x = 1$, i.e. $x = 1/2 \notin N$. So 1 has no pre-image. Hence f is NOT onto.
Step Conclude precisely [0.5 mark]	**Conclude**	Therefore $f : N \to N$, $f(x) = 2x$ is one-one but not onto. Hence not bijective.

COMMON LOSS OF MARKS:

- Forgetting to state the definition before using it — examiner deducts 0.5 marks.
- Using a numerical example to 'prove' one-one (examples never prove, they only disprove).
- Saying 'odd numbers are not hit' without naming a specific y that has no pre-image.

★ TOPPER TEMPLATE — Composition / algebra of functions at a point (2 marks)

6 out of every 10 papers

Step Write out f(x) and g(x) cleanly [0.25 mark]	**Setup**	$f(x) = 2x + 1$, $g(x) = x^2$.
Step Compute the inner function value first [0.75 mark]	**Inner**	$(f \circ g)(2)$: compute $g(2)$ first. $g(2) = 2^2 = 4$.
Step Substitute into the outer function [0.75 mark]	**Outer**	$f(g(2)) = f(4) = 2 \cdot 4 + 1 = 9$. Hence $(f \circ g)(2) = 9$.
Step Add a one-line remark on non-commutativity if 2-part question [0.25 mark]	**Remark**	Similarly $(g \circ f)(2) = g(5) = 25 \neq 9$, confirming $f \circ g \neq g \circ f$.

COMMON LOSS OF MARKS:

- Computing $f(2)$ instead of $g(2)$ first — order error in composition.
- Skipping the substitution step; examiner can't follow the working.
- Treating $(f \circ g)$ and $(g \circ f)$ as equal — a classical trap.

★ TOPPER TEMPLATE — Sketch a standard function ([x], |x|, signum) and state domain/range (4 marks)

5 out of every 10 papers

Step Make a small table of values across the relevant interval **[1.0 mark]**	**Table**	For f(x) = [x] on [−2, 3): x ∈ [−2,−1) ⇒ f = −2; x ∈ [−1,0) ⇒ f = −1; x ∈ [0,1) ⇒ f = 0; x ∈ [1,2) ⇒ f = 1; x ∈ [2,3) ⇒ f = 2.
Step Draw a labelled axes diagram with the staircase steps **[1.5 marks]**	**Sketch**	Mark x-axis from −3 to 4, y-axis from −3 to 3. Draw 5 horizontal segments at y = −2, −1, 0, 1, 2. Filled dot at the LEFT end of each step, open circle at the RIGHT end.
Step State domain on the asked interval **[0.5 mark]**	**Domain**	Domain (on given interval) = [−2, 3).
Step State range using only the y-values actually hit **[1.0 mark]**	**Range**	Range = {−2, −1, 0, 1, 2}.

COMMON LOSS OF MARKS:

- Connecting the steps with diagonal lines — staircase is DISCONTINUOUS.
- Open/filled dot reversed (filled goes on the closed end of each interval).
- Stating range as an interval [−2, 2] instead of the discrete set {−2, −1, 0, 1, 2}.

MARKING SCHEME — GENERAL NOTES

- Section A: full mark only for the final numerical/symbolic answer in correct form; no partial marks.
- Section B: 0.5 mark for stating the method/condition, 1 mark for correct working, 0.5 for final answer in required notation.
- Section C: split as 0.5 for definition/condition + 1.5 for proof/solution + 1 for final answer. Use of an example to 'prove' a general statement loses 1 mark.
- Section D: graphical questions — 2 marks for correctly labelled sketch with axes, 1 for filled/open dots correct (where applicable), 1 for stated range/domain. Algebraic questions in this section split as 1 + 1 + 1 + 1 across the sub-parts.
- Mention of domain in interval notation (e.g. [−3, 3]) accepted in place of set-builder.
- Confusing range with codomain → −1 mark (cumulative across paper).
- Greatest integer staircase joined by diagonals (instead of discrete steps) → −1 mark.
- Stating (f∘g)(x) = (g∘f)(x) without verification → automatic loss of 1 mark on composition questions.