ANSWER KEY & MARKING SCHEME · CBSE CLASS 11

Trigonometric Functions

Mathematics · Chapter 3 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Very Short Answer

Q1. Convert 45° into radians. [1 mark]

Ans: $45^\circ \times (\pi/180) = \pi/4$ rad.

Q2. Find the value of $\sin(-30^\circ)$. [1 mark]

Ans: sin is odd: $\sin(-30^\circ) = -\sin 30^\circ = -1/2$.

Q3. Write the period of the function $f(x) = \cos 2x$. [1 mark]

Ans: Period of $\cos(kx) = 2\pi/|k|$. Here $k = 2$, period $= \pi$.

Q4. State the range of the function $f(x) = \sin x$. [1 mark]

Ans: Range $= [-1, 1]$.

Q5. Evaluate $\cos(\pi/2 - \pi/3)$. [1 mark]

Ans: $\cos(\pi/2 - \pi/3) = \sin(\pi/3) = \sqrt{3}/2$.

Section B — Short Answer

Q6. Find the length of an arc of a circle of radius 8 cm subtending a central angle of $3\pi/4$ rad. [2 marks]

Ans: $s = r\theta = 8 \times 3\pi/4 = 6\pi$ cm ≈ 18.85 cm.

Q7. If $\sin x = 3/5$ and x lies in Q2, find cos x and tan x. [2 marks]

Ans: $\cos^2 x = 1 - 9/25 = 16/25 \Rightarrow |\cos x| = 4/5$. In Q2 cos is negative $\Rightarrow \cos x = -4/5$. $\tan x = \sin x / \cos x = (3/5)/(-4/5) = -3/4$.

Q8. Find the area of a sector of radius 5 cm with a central angle of $\pi/6$ rad. [2 marks]

Ans: $A = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 25 \times \pi/6 = 25\pi/12$ cm² ≈ 6.54 cm².

Q9. Evaluate $\cos 15^\circ$ using a sum/difference formula. [2 marks]

Ans: $\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ = (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) = (\sqrt{3} + 1)/(2\sqrt{2}) = (\sqrt{6} + \sqrt{2})/4$.

Section C — Long Answer

Q10. Prove that $(\cos 7x + \cos 5x)/(\sin 7x - \sin 5x) = \cot x$. [3 marks]

Ans: Numerator: $\cos 7x + \cos 5x = 2 \cos 6x \cos x$ (sum-to-product). Denominator: $\sin 7x - \sin 5x = 2 \cos 6x \sin x$. Ratio $= \cos x / \sin x = \cot x$ = RHS. ✓

Q11. Find the general solution of $\sin 2x = \sqrt{3}/2$. [3 marks]

Ans: $\sqrt{3}/2 = \sin(\pi/3)$. So $\sin 2x = \sin(\pi/3)$. General solution: $2x = n\pi + (-1)^n(\pi/3) \Rightarrow x = n\pi/2 + (-1)^n(\pi/6)$, $n \in \mathbb{Z}$.

Q12. If $\tan x = 5/12$ and x lies in Q3, find sin x, cos x, sec x, cosec x. [3 marks]

Ans: $1 + \tan^2 x = \sec^2 x = 1 + 25/144 = 169/144 \Rightarrow |\sec x| = 13/12$. In Q3 cos (hence sec) is negative $\Rightarrow \sec x = -13/12$ and $\cos x = -12/13$. $\sin x = \tan x \cdot \cos x = (5/12)(-12/13) = -5/13$. $\operatorname{cosec} x = 1/\sin x = -13/5$.

Section D — Application

Q13. **(a) Evaluate sin 75° using a sum formula. (b) Evaluate cos 105° using a sum formula. (c) Hence show that sin 75° + cos 105° equals (√6 + √2 − √6 + √2)/4 = (2√2)/4 = √2/2, and state which standard angle equals √2/2.** **[4 marks]**

> **Ans:** (a) sin 75° = sin(45° + 30°) = sin 45° cos 30° + cos 45° sin 30° = (√6 + √2)/4. (b) cos 105° = cos(60° + 45°) = cos 60° cos 45° − sin 60° sin 45° = (1/2)(1/√2) − (√3/2)(1/√2) = (1 − √3)/(2√2) = (√2 − √6)/4. (c) sin 75° + cos 105° = (√6 + √2)/4 + (√2 − √6)/4 = (2√2)/4 = √2/2 = sin 45° = cos 45°.

Q14. **A horse is tethered by a rope of length 14 m to a peg at one corner of a rectangular field. The horse grazes a quarter-circle region. (a) Find the angle of the sector in radians. (b) Find the length of the arc the horse traces along the outer edge of its grazing region. (c) Find the area the horse can graze.** **[4 marks]**

> **Ans:** (a) A quarter circle subtends 90° = $\pi/2$ rad at the centre. (b) Arc length $s = r\theta = 14 \times \pi/2 = 7\pi$ m ≈ 21.99 m. (c) Sector area $A = ½ r^2 \theta = ½ \times 196 \times \pi/2 = 49\pi$ m² ≈ 153.94 m². (Sanity check: a quarter of $\pi r^2 = \pi \cdot 196/4 = 49\pi$ ✓.)

★ TOPPER ANSWER TEMPLATES

5 TEMPLATES · MEMORISE THE FORMAT

★ TOPPER TEMPLATE — Convert degrees ↔ radians / arc-length / sector area (2 marks)

Asked in roughly 9 out of every 10 CBSE papers

Step State the conversion factor and convert the given angle [0.5 mark]	**Convert**	Given θ = 15°. 1° = π/180 rad ⇒ θ = 15 × π/180 = π/12 rad.
Step Write the formula you'll use with units check [0.5 mark]	**Formula**	Arc length $s = r\theta$ (θ in radians). Here r = 5 cm.
Step Substitute and compute [0.5 mark]	**Substitute**	s = 5 × π/12 = 5π/12 cm.
Step Report with units (and decimal if asked) [0.5 mark]	**Answer**	s ≈ 5 × 3.1416 / 12 ≈ 1.31 cm. Final answer: 5π/12 cm ≈ 1.31 cm.

COMMON LOSS OF MARKS:

- Plugging θ = 15 (degrees) directly into $s = r\theta$ — gives wrong arc length.
- Forgetting units (cm).
- Mixing up $s = r\theta$ (arc length) with $A = ½r^2\theta$ (sector area).

★ TOPPER TEMPLATE — Find remaining trig ratios from one ratio + quadrant (3 marks)

8 out of every 10 papers

Step Quote $\sin^2+\cos^2=1$ and substitute the known value [0.5 mark]	**Identity**	Given sin x = 3/5. Use $\sin^2 x + \cos^2 x = 1 \Rightarrow 9/25 + \cos^2 x = 1 \Rightarrow \cos^2 x = 16/25$.
Step Take the positive square root and then fix sign using quadrant [1.0 mark]	**Square root + ASTC**	$\lvert\cos x\rvert = 4/5$. x is in Q2 where cos is NEGATIVE (ASTC: only sin positive in Q2). Hence cos x = −4/5.
Step Build tan, cot, sec, cosec from sin and cos [1.0 mark]	**Build**	tan x = sin x / cos x = (3/5)/(−4/5) = −3/4. cot x = −4/3. sec x = 1/cos x = −5/4. cosec x = 1/sin x = 5/3.
Step Box / list final answers [0.5 mark]	**State**	cos x = −4/5, tan x = −3/4, cot x = −4/3, sec x = −5/4, cosec x = 5/3.

COMMON LOSS OF MARKS:

- Choosing the wrong sign for cos (ignored quadrant) — automatic −1 mark.
- Forgetting reciprocal ratios cosec / sec / cot.
- Writing tan x = +3/4 because both signs were dropped in squaring.

★ TOPPER TEMPLATE — Prove a trigonometric identity (3 marks)

7 out of every 10 papers

Step State which side you'll start from and why [0.5 mark]	**Plan**	Start from LHS = (cos 7x + cos 5x)/(sin 7x − sin 5x), the more complex side, and reduce to cot x.
Step Apply sum-to-product formulas to numerator and denominator [1.5 marks]	**Transform**	cos C + cos D = 2 cos((C+D)/2) cos((C−D)/2). So num = 2 cos 6x cos x. sin C − sin D = 2 cos((C+D)/2) sin((C−D)/2). So den = 2 cos 6x sin x.
Step Cancel common factor and conclude [1.0 mark]	**Simplify**	LHS = (2 cos 6x cos x)/(2 cos 6x sin x) = cos x / sin x = cot x = RHS. ∴ LHS = RHS, proved.

COMMON LOSS OF MARKS:

- Cancelling cos 6x without noting cos 6x ≠ 0 — small but examiners can deduct 0.5.
- Mixing the sum-to-product formula with the product-to-sum formula.
- Working on both sides simultaneously without a justification — examiner counts it as 'circular'.

★ TOPPER TEMPLATE — General solution of a trig equation (3 marks)

6 out of every 10 papers

Step Reduce equation to standard form sin x = sin α / cos x = cos α / tan x = tan α [0.5 mark]	**Reduce**	$\sin 2x = \sqrt{3}/2 = \sin(\pi/3)$. So $\sin 2x = \sin(\pi/3)$.
Step Apply the correct general-solution formula [1.5 marks]	**Formula**	For sin form: $2x = n\pi + (-1)^n \cdot (\pi/3)$, $n \in Z$.
Step Solve for x [0.75 mark]	**Solve**	$x = n\pi/2 + (-1)^n \cdot (\pi/6)$, $n \in Z$.
Step State n ∈ Z explicitly [0.25 mark]	**Domain of n**	Where n is any integer.

COMMON LOSS OF MARKS:

- Using the cos rule ($2n\pi \pm \alpha$) for a sin equation — biggest single-mark mistake.
- Dropping the $(-1)^n$ factor.
- Not stating $n \in Z$ (the marker treats the answer as incomplete).

★ TOPPER TEMPLATE — Evaluate sin 75°, cos 15°, tan 105° using sum/difference (4 marks)

5 out of every 10 papers

Step Decompose the unknown angle as a sum of standard angles **[1.0 mark]**	**Decompose**	$75° = 45° + 30°$.
Step Quote sum formula and substitute standard values **[1.5 marks]**	**Apply formula**	$\sin(A+B) = \sin A \cos B + \cos A \sin B$. $\sin 75° = \sin 45° \cos 30° + \cos 45° \sin 30° = (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2)$.
Step Combine fractions **[1.0 mark]**	**Combine**	$= (\sqrt{3} + 1)/(2\sqrt{2})$.
Step Rationalise the denominator and state the boxed answer **[0.5 mark]**	**Rationalise**	$= (\sqrt{3} + 1)/(2\sqrt{2}) \times \sqrt{2}/\sqrt{2} = (\sqrt{6} + \sqrt{2})/4$.

COMMON LOSS OF MARKS:

- Using sin(A–B) when sum was needed (or vice versa).
- Forgetting standard values ($\sin 30° = 1/2$, $\cos 45° = 1/\sqrt{2}$ etc.).
- Leaving answer un-rationalised — examiner deducts 0.5 in board marking.

MARKING SCHEME — GENERAL NOTES

- Section A: full mark only for the final numerical/symbolic answer in correct form; no partial marks.
- Section B: 0.5 for stating the formula, 1 for clean working, 0.5 for the final boxed answer (with units where applicable).
- Section C: split as 0.5 for stating identity / formula + 1.5 for transformation/working + 1 for final answer or conclusion. An identity 'proof' that works both sides simultaneously without justification loses 1 mark.
- Section D: split as 1 + 1 + 1 + 1 across sub-parts; correct units (cm, m, rad) carry 0.5 of the final-answer mark in each sub-part.
- θ in $s = r\theta$ or $A = ½r^2\theta$ must be in radians. Treating θ as degrees → automatic −1 mark.
- Wrong quadrant sign (e.g. cos positive in Q2) → −1 mark.
- Using general solution of cos ($2n\pi \pm \alpha$) for a sin equation, or vice versa → −1 mark.
- Final answers not rationalised (e.g. left as $(\sqrt{3}+1)/(2\sqrt{2})$) → −0.5 mark in board marking.