Ready For Boards
10th & 12th Exam Prep

CHAPTER 1

# Numbers, Quantification & Numerical Applications

*CBSE Class 12 Applied Mathematics · Unit 1*

CBSE · Applied Mathematics · Class 12

## WHAT THIS CHAPTER DOES

**A** — Memorise the four headline formulas — allegation, boats-streams, pipes-rate, inequality-flip — by the end of this lesson.

**B** — Translate at least one PYQ from each of five topics into its mathematical form within 30 seconds of reading the question.

*Boards prep that builds confidence, not anxiety.*

TODAY'S MISSION

# Today's mission

1. Memorise the four headline formulas — allegation, boats-streams, pipes-rate, inequality-flip — by the end of this lesson.
2. Translate at least one PYQ from each of five topics into its mathematical form within 30 seconds of reading the question.
3. Solve a 4-mark allegation problem using the criss-cross diagram AND verify with a weighted-average check.
4. Solve a numerical inequality and interpret the answer in plain English.

WHY THIS MATTERS

# Why this chapter matters

1. Opens Class 12 Applied Maths. Every later unit assumes you can translate a real-world situation into a numerical equation.

2. 4-6 marks per board paper. Allegation + boats-streams + inequality together account for ~70% of those marks.

3. Outside the exam: every commerce-stream student will use these techniques as adults — mixing products, calculating combined rates, solving inequalities under business constraints.

TOPIC

# A

# Modulo arithmetic

THEOREM · LOAD-BEARING RESULT

# Congruence modulo n — the load-bearing definition

> *Two integers x and a are said to be congruent modulo n, written $x \equiv a \pmod{n}$, if and only if (x − a) is divisible by n — equivalently, x and a leave the same remainder when divided by n.*

### STATEMENT

$x \equiv a \pmod{n} \Leftrightarrow n \mid (x - a) \Leftrightarrow x = a + kn$ for some integer k $\Leftrightarrow x \bmod n = a \bmod n$. Worked example: $23 \equiv 3 \pmod{5}$ because $23 - 3 = 20$ is divisible by 5; or because both 23 and 3 leave the same remainder (3) when divided by 5.

### WHY THIS MATTERS

- Modulo is the mathematical tool behind cyclic phenomena — days of the week (mod 7), hours of a clock (mod 12 or 24), even/odd (mod 2), divisibility tests
- Once a student sees the modulo behind these, all day-of-week and clock problems become two-line solutions.

### WATCH OUT FOR

**NOTE** Do NOT read $x \equiv a \pmod{n}$ as 'x divided by n equals a'. The symbol ≡ means SAME REMAINDER — not division. The single most common error in modulo questions is treating the symbol as ordinary equality.

WORKED EXAMPLE

# Worked example — day after 100 days

1. QUESTION: If today is Wednesday, what day of the week will it be after 100 days?
2. STEP 1 — Translate to mod 7: A week has 7 days. We need 100 mod 7. Divide: $100 = 7 \times 14 + 2$, so $100 \equiv 2 \pmod 7$. After 100 days, the day is TWO weekdays beyond Wednesday.
3. STEP 2 — Count two days from Wednesday: Thursday (1), Friday (2). Therefore, after 100 days, the day will be FRIDAY.
4. STEP 3 — Mental check: $7 \times 14 = 98$ days = exactly 14 weeks → same day (Wednesday). 100 days = 98 + 2 = two days beyond Wednesday = Friday. ✓
5. TAKEAWAY: For ANY 'what day after N days' problem, the answer is N mod 7 days forward. The hardest part of the question is computing the remainder cleanly.

TOPIC

# B

# Allegation and mixture

THEOREM · LOAD-BEARING RESULT

# The allegation rule — exchange of differences

> *When two ingredients of cost-prices C (cheaper) and D (dearer) are mixed to produce a mixture of mean-price M (with $C < M < D$), the ratio in which they must be mixed is (cheaper : dearer) = $(D - M) : (M - C)$.*

### STATEMENT

**Rule: ratio of cheaper to dearer quantity = $(D - M) : (M - C)$. The differences $(D - M)$ and $(M - C)$ exchange sides — the difference involving the DEARER price goes to the CHEAPER side of the ratio, and vice-versa. Visualise this as a**

### WHY THIS MATTERS

- Allegation is the most-asked topic in this chapter (~24% of marks)
- It generalises to ANY situation where two quantities at different rates combine to give a known combined rate — milk + water mixtures, alloys, blended teas, average-speed problems
- Once the criss-cross is automatic, the whole topic becomes mechanical

### WATCH OUT FOR

**NOTE** The differences EXCHANGE sides — this is the most-mistaken step. The difference $(D - M)$, involving the dearer price, ends up on the CHEAPER side of the ratio. Drawing the criss-cross diagram every single time prevents this error.

WORKED EXAMPLE

# Worked example — tea-blending problem

1. QUESTION: A shopkeeper mixes two varieties of tea costing ₹350/kg and ₹500/kg to obtain a mixture costing ₹430/kg. In what ratio should they be mixed?

2. STEP 1 — Identify: C (cheaper) = ₹350. D (dearer) = ₹500. M (mean) = ₹430. Sanity check — M lies between C and D: $350 < 430 < 500$ ✓.

3. STEP 2 — Apply the allegation rule: (cheaper : dearer) = (D − M) : (M − C) = (500 − 430) : (430 − 350) = 70 : 80 = 7 : 8.

4. STEP 3 — Verify with a weighted-average check: 7 kg at ₹350 = ₹2,450; 8 kg at ₹500 = ₹4,000. Total: 15 kg at total cost ₹6,450. Per kg: 6,450 ÷ 15 = ₹430. ✓ Matches the stated mean.

5. ANSWER: Mix in the ratio 7 : 8 (cheaper : dearer). The verification step earns the final mark and catches sign-errors before they reach the answer line.

TOPIC

# C

## Boats, streams, pipes

THEOREM · LOAD-BEARING RESULT

# Boats and streams — the two-formula skeleton

> *If b is the speed of the boat in still water and s is the speed of the stream, then downstream speed = (b + s) and upstream speed = (b − s). Inverting, the boat's still-water speed is the AVERAGE of downstream and upstream, and the stream's speed is HALF the DIFFERENCE.*

### STATEMENT

**Definitions: downstream speed = (b + s); upstream speed = (b − s). Inverting: b = (downstream + upstream) ÷ 2; s = (downstream − upstream) ÷ 2. The stream HELPS the boat going downstream (faster) and OPPOSES it going**

### WHY THIS MATTERS

- Boats-streams problems are PYQ-staple — featured in roughly 85% of board papers
- Once the four-formula skeleton is internalised, every such problem solves in two lines: compute the two speeds from the given times, apply b = (d + u)/2 and s = (d − u)/2.

### WATCH OUT FOR

**NOTE** Do NOT confuse the direction signs. DOWNSTREAM is FASTER (boat + stream). UPSTREAM is SLOWER (boat − stream). Reversing this is the most common 2-mark loss in this chapter. The English-language direction matters — 'going with the current' = downstream = faster.

TOPIC

# Pipes and cisterns — always work in RATES

### TRANSLATE EVERY PIPE TO A RATE

A pipe that fills a tank in T hours has a fill-rate of (1/T) of the tank per hour — this rate framing is the single most important conceptual move in the entire topic. A leak that empties a full tank in T hours has rate (−1/T) per hour; the minus sign is critical because leaks are mathematically just negative-rate filling pipes.

### COMBINED RATE = SUM OF INDIVIDUAL RATES

Once pipes and leaks are in rate form, combined rate is just the algebraic sum of individual rates — exactly how forces add in physics (vectors) or how currents add at a node in electricity (Kirchhoff). Worked example: pipe A fills at +1/4 per hour and a leak empties at −1/12 per hour; combined rate = 1/4 − 1/12 = 3/12 −

### TIME = 1 / COMBINED RATE

From a combined rate of 1/6 tank per hour, the time to fill the full tank is 1 ÷ (1/6) = 6 hours — the reciprocal of the rate. ALWAYS invert at the end; this is the single most-forgotten step in the topic and the reason students who set up the rates correctly still lose 1-2 marks on the final answer. If the combined rate works out negative, the tank is

### WHY AVERAGING TIMES IS WRONG

The single most common student error: averaging individual times to get the combined time. Beginners write 'combined time = (4 + 12)/2 = 8 hours' — but this is mathematically meaningless because times cannot be averaged for parallel processes. The correct rule, derivable from first principles, is: RATES

TOPIC

# D

# Numerical inequalities

THEOREM · LOAD-BEARING RESULT

# The sign-flip rule — the load-bearing skill of inequalities

> *When you multiply or divide both sides of an inequality by a POSITIVE number, the inequality sign stays the SAME. When you multiply or divide both sides by a NEGATIVE number, the inequality sign FLIPS.*

### STATEMENT

**Positive multiplier: $a < b \Rightarrow ka < kb$ for $k > 0$. Negative multiplier: $a < b \Rightarrow ka > kb$ for $k < 0$. Worked example: from $-2x > 6$, dividing both sides by $-2$ (negative) FLIPS the sign: $x < -3$. From $-2x > 6$, dividing by $+2$ keeps the sign: $-x >$**

### WHY THIS MATTERS

- Examiners deliberately design inequality questions so that the student MUST divide by a negative coefficient to reach the answer
- Students who forget to flip get the answer with the wrong direction — and lose the full mark
- Mastering this rule on its own is worth 4 marks per paper

### WATCH OUT FOR

**NOTE** ADDING or SUBTRACTING the same value on both sides does NOT change the sign — the sign-flip applies ONLY to multiplication and division. Don't flip when adding.

TOPIC

# Real-world inequality questions — three common patterns

## PRODUCTION CONSTRAINT

Production constraints arise when a finite resource (labour-hours, machine-time, raw material) caps how much output can be produced. Worked example: a manufacturer has 500 labour-hours available per week, and each unit of output requires 2 hours; the inequality is $2x \le 500$, solving to $x \le 250$ units. The same template handles

## BUDGET CONSTRAINT

Budget constraints arise when a fixed monetary budget must be allocated across two or more items with different unit prices. Worked example: a student has ₹400 to buy books at ₹120 each and notebooks at ₹40 each; the inequality is $120b + 40n \le 400$. Combine this with the natural constraints $b \ge 0$ and $n \ge 0$ (purchases must be non-

## PROFIT THRESHOLD

Profit-threshold inequalities flip the direction — instead of capping a maximum (≤), they demand a minimum performance (≥). Worked example: a vendor wants profit at least ₹2,000 per day, and profit per item sold is ₹150; the inequality is $150x \ge 2{,}000$, solving to $x \ge 13.33$. Since x must be an integer (you can't sell a third of an item), round UP

## HOW TO TRANSLATE THE WORDS

Translating English-language constraints into mathematical inequalities is half the question — get the direction wrong and the entire downstream solution is wasted effort. The canonical dictionary: 'at most' and 'no more than' map to ≤; 'at least' and 'no less than' map to ≥; 'more than' and 'exceeds' map to

TOPIC

# Modulo congruence

TRAP → TRUTH

× **MISTAKE** x ≡ a (mod n) means x divided by n equals a.

✓ **CORRECT** x ≡ a (mod n) means x AND a leave the SAME REMAINDER when divided by n — equivalently, (x − a) is divisible by n. Example: 23 ≡ 3 (mod 5) because (23 − 3) = 20 is divisible by 5; or because both 23 and 3 leave remainder 3 when divided by 5. Reading the symbol as 'equals' is the single biggest cause of wrong-answer modulo questions.

TOPIC

# Allegation — rule of cross multiplication

TRAP → TRUTH

× **MISTAKE** In allegation, the ratio is given by (higher price : lower price).

✓ **CORRECT** The allegation rule is: ratio of CHEAPER quantity to DEARER quantity = (DEARER price − MEAN price) : (MEAN price − CHEAPER price). The ratio is built from the DIFFERENCES with the mean, not from the absolute prices. Visualise it as the criss-cross — the differences exchange sides.

TOPIC

# Boats and streams — direction confusion

TRAP → TRUTH

× **MISTAKE** Downstream speed = boat speed − stream speed.

✓ **CORRECT** Downstream speed = (boat speed) + (stream speed) — the stream HELPS the boat. Upstream speed = (boat speed) − (stream speed) — the stream OPPOSES the boat. Hence: boat speed in still water = (downstream + upstream) ÷ 2, and stream speed = (downstream − upstream) ÷ 2. Swapping the signs is the most common 2-mark loss in this chapter.

TOPIC

# Pipes and cisterns — leaks treated like inlets

TRAP → TRUTH

× **MISTAKE** If pipe A fills in 6 hours and a leak empties in 12 hours, the combined fill time is (6 + 12)/2 = 9 hours.

✓ **CORRECT** Convert each pipe to a RATE: A fills at +1/6 tank per hour; leak empties at −1/12 tank per hour. Combined rate = 1/6 − 1/12 = 1/12 tank per hour → tank fills in 12 hours. The mistake is averaging times instead of summing rates. Always work in RATES, not in times.

TOPIC

# Numerical inequality — flipping the sign on multiplication

TRAP → TRUTH

× **MISTAKE** When you multiply or divide both sides of an inequality by any number, the inequality sign stays the same.

✓ **CORRECT** If you multiply or divide both sides of an inequality by a POSITIVE number, the sign STAYS the same. If you multiply or divide both sides by a NEGATIVE number, the sign FLIPS. Example: from $-2x > 6$, dividing by $-2$ gives $x < -3$ (sign flipped). This is the single most-tested skill on inequality questions — examiners deliberately design problems where the student must divide by a negative coefficient.

TOPPER TEMPLATE · MARK-BY-MARK

# 3 marks: 'If today is Monday, what day will it be after 80 days?'

| Step | Task | Marks | Explanation |
|---|---|---|---|
| 1 | **TRANSLATE TO MODULAR FORM** | 1 m | There are 7 days in a week. The day after N days is found by computing N mod 7 and counting that many days forward from today. We need 80 mod 7. |
| 2 | **COMPUTE THE REMAINDER** | 1 m | Divide 80 by 7: $7 \times 11 = 77$, remainder $= 80 - 77 = 3$. Hence $80 \equiv 3 \pmod 7$. That is, after 80 days, the day is 3 weekdays beyond Monday. |
| 3 | **STATE THE ANSWER IN WORDS** | 1 m | Counting 3 days from Monday: Tuesday (1), Wednesday (2), Thursday (3). Therefore, after 80 days, the day will be THURSDAY. Note that the question would have a different answer if it asked for 80 days BEFORE Monday — in that case count backwards: Sunday, Saturday, Friday. |

TOPPER TEMPLATE · MARK-BY-MARK

# 4 marks: 'A shopkeeper mixes two varieties of tea costing ₹350/kg and ₹500/kg to obtain a mixture costing ₹430/kg. In

| Step | Task | Marks | Working |
|---|---|---|---|
| 1 | **IDENTIFY CHEAPER, DEARER, MEAN** | 1 m | Cheaper price (C) = ₹350/kg. Dearer price (D) = ₹500/kg. Mean price (M) = ₹430/kg. Confirm M lies between C and D: $350 < 430 < 500$ ✓. (If M were outside this range, no positive mixture would exist.) |
| 2 | **APPLY THE ALLEGATION RULE** | 1 m | Allegation rule: (quantity of cheaper) : (quantity of dearer) = (D − M) : (M − C). Substituting: (500 − 430) : (430 − 350) = 70 : 80 = 7 : 8. |
| 3 | **VERIFY WITH A CHECK** | 1 m | Quick verification: take 7 kg cheaper (cost = 7 × 350 = ₹2450) + 8 kg dearer (cost = 8 × 500 = ₹4000). Total = 15 kg costing ₹6450. Per kg = 6450 ÷ 15 = ₹430. ✓ Matches the stated mean — ratio is correct. |
| 4 | **STATE THE ANSWER WITH UNITS** | 1 m | The two varieties must be mixed in the ratio 7 : 8 (cheaper : dearer). For every 7 kg of ₹350 tea, the shopkeeper must add 8 kg of ₹500 tea to obtain a mixture costing ₹430/kg. |

TOPPER TEMPLATE · MARK-BY-MARK

# 3 marks: 'A boat covers 24 km downstream in 2 hours and the same distance upstream in 3 hours. Find the speed of the

| | | |
|---|---|---|
| 1 | **COMPUTE DOWNSTREAM AND UPSTREAM SPEEDS** 1 m | Downstream speed = distance ÷ time = 24 ÷ 2 = 12 km/h. Upstream speed = 24 ÷ 3 = 8 km/h. Recall: downstream speed = (boat + stream); upstream speed = (boat − stream). |
| 2 | **APPLY THE FORMULAS** 1 m | Speed of boat in still water = (downstream + upstream) ÷ 2 = (12 + 8) ÷ 2 = 10 km/h. Speed of the stream = (downstream − upstream) ÷ 2 = (12 − 8) ÷ 2 = 2 km/h. |
| 3 | **STATE AND CHECK** 1 m | Therefore, the boat's speed in still water is 10 km/h and the stream's speed is 2 km/h. CHECK: 10 + 2 = 12 ✓ (matches downstream); 10 − 2 = 8 ✓ (matches upstream). Both consistency checks pass. |

PYQ PATTERNS

# Top PYQ patterns to drill

| | | |
|---|---|---|
| #1 | Find x such that x ≡ a (mod n) given a numerical situation (clock, calendar, day-of-week). (2 marks) | Annual |
| #2 | Allegation: a shopkeeper mixes two grades of rice / two qualities of tea at given prices to obtain a mixture at a stated price | find the ratio. (3 marks) — Annual |
| #3 | Boats and streams: given downstream + upstream speeds, find the speed of the boat in still water and the speed of the stream. (3 marks) | Most years |
| #4 | Pipes and cisterns: two pipes, with one a leak, find time to fill a tank. (3-4 marks) | 2019, 2021, 2023 |
| #5 | Solve a numerical inequality arising from a real-world constraint (a manufacturer's production / a budget) and interpret the solution. (4 marks) | 2020, 2022, 2024 |

MARKS DISTRIBUTION

# 10-year marks distribution

**10-YEAR PYQ MARKS DISTRIBUTION**

| | |
|---|---|
| Modulo arithmetic + congruence applications | 22% |
| Allegation + mixture | 24% |
| Boats and streams · pipes and cisterns | 18% |
| Races and games · partnership | 12% |
| Numerical inequalities — real-world application | 16% |
| Mixed numerical / multi-step problems | 8% |

RECAP · MEMORISE THESE

# Recap — the four headline formulas

**1** Modulo — x ≡ a (mod n) means (x − a) is divisible by n. Day after N days = (today + N mod 7).

**2** Allegation — (cheaper : dearer) = (D − M) : (M − C). Differences exchange sides. Always verify with weighted-average.

**3** Boats and streams — b = (down + up) ÷ 2; s = (down − up) ÷ 2. Downstream is faster (with the current); upstream is slower.

**4** Pipes and cisterns — Work in RATES (1/T per unit time). Combined rate = sum of individuals (leaks are negative).

**5** Inequality sign-flip — Multiply/divide by NEGATIVE → FLIP the sign. Add/subtract → sign unchanged.

WHAT'S NEXT

# What's next

- → Chapter 2 — Algebra (Matrices: introduction, operations, transpose).
- → Sit the 15-MCQ Quick Drill for this chapter.
- → Then the full Board-Pattern Paper — 30 marks.

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# You've mastered Unit 1 of Applied Maths.

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