ANSWER KEY & MARKING SCHEME · CBSE CLASS 12

Algebra (Matrices and Determinants)

Applied Mathematics · Chapter 2 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Short calculation (2 × 3 = 6 marks)

Q1. If [[2x, 5],[7, y−3]] = [[8, 5],[7, 4]], find the values of x and y. [2 marks]

Ans: Equate corresponding entries: $2x = 8 \Rightarrow x = 4$; $y - 3 = 4 \Rightarrow y = 7$. Hence $x = 4$, $y = 7$.

Q2. Evaluate the determinant |[[4,3],[2,5]]|. [2 marks]

Ans: $|A| = (4)(5) - (3)(2) = 20 - 6 = 14$.

Q3. If A = [[1,2],[3,4]], find 3A − 2I, where I is the 2×2 identity matrix. [2 marks]

Ans: 3A = [[3,6],[9,12]]; 2I = [[2,0],[0,2]]; 3A − 2I = [[3−2, 6−0],[9−0, 12−2]] = [[1,6],[9,10]].

Section B — Problems with full working (3 × 4 = 12 marks)

Q4. Express A = [[1,3],[5,7]] as the sum of a symmetric and a skew-symmetric matrix. [3 marks]

Ans: A' = [[1,5],[3,7]]. Symmetric P = (1/2)(A+A') = (1/2)[[2,8],[8,14]] = [[1,4],[4,7]]. Skew Q = (1/2)(A−A') = (1/2)[[0,−2],[2,0]] = [[0,−1],[1,0]]. Check P+Q = [[1,3],[5,7]] = A ✓; diagonal of Q is zero ✓.

Q5. If A = [[2,1],[3,2]] and B = [[1,0],[1,1]], find AB. Is AB = BA? Justify. [3 marks]

Ans: AB = [[2·1+1·1, 2·0+1·1],[3·1+2·1, 3·0+2·1]] = [[3,1],[5,2]]. BA = [[1·2+0·3, 1·1+0·2],[1·2+1·3, 1·1+1·2]] = [[2,1],[5,3]]. Since [[3,1],[5,2]] ≠ [[2,1],[5,3]], AB ≠ BA — matrix multiplication is not commutative.

Q6. Find the inverse of A = [[2,3],[1,4]] using the adjoint method. [3 marks]

Ans: $|A| = (2)(4) - (3)(1) = 8 - 3 = 5 \neq 0$, so A^{-1} exists. adj(A) = [[4,−3],[−1,2]]. A^{-1} = (1/5)[[4,−3],[−1,2]] = [[4/5, −3/5],[−1/5, 2/5]]. Check: $A \cdot A^{-1}$ = (1/5)[[5,0],[0,5]] = I ✓.

Q7. Evaluate the determinant |[[1,2,3],[4,5,6],[7,8,10]]| by expanding along the first row. [3 marks]

Ans: $|A| = 1\cdot(5\cdot10 - 6\cdot8) - 2\cdot(4\cdot10 - 6\cdot7) + 3\cdot(4\cdot8 - 5\cdot7) = 1\cdot(50-48) - 2\cdot(40-42) + 3\cdot(32-35) = 1\cdot2 - 2\cdot(-2) + 3\cdot(-3) = 2 + 4 - 9 = -3$.

Section C — Long-answer (6 × 2 = 12 marks)

Q8. Solve the following system of equations by the matrix (inverse) method: 2x + 3y = 13 and x + y = 5. Verify your solution in both equations. [6 marks]

Ans: Write AX = B with A = [[2,3],[1,1]], B = [[13],[5]]. $|A| = (2)(1) - (3)(1) = -1 \neq 0$, so a unique solution exists. adj(A) = [[1,−3],[−1,2]]; A^{-1} = (1/−1)[[1,−3],[−1,2]] = [[−1,3],[1,−2]]. $X = A^{-1}B$ = [[(−1)(13)+(3)(5)],[(1)(13)+(−2)(5)]] = [[−13+15],[13−10]] = [[2],[3]]. So x = 2, y = 3. Verify: 2(2)+3(3) = 4+9 = 13 ✓; 2+3 = 5 ✓.

Q9. A shopkeeper sells two products. The cost matrix and quantity matrix give the total cost via matrix multiplication. Given and the system arising as 3x + 2y = 12, x + y = 5, solve for x and y by the matrix method, and state the value of x + y. [6 marks]

Ans: Write AX = B with A = [[3,2],[1,1]], B = [[12],[5]]. $|A| = (3)(1) - (2)(1) = 1 \neq 0$, so a unique solution exists. adj(A) = [[1,−2],[−1,3]]; A^{-1} = (1/1)[[1,−2],[−1,3]] = [[1,−2],[−1,3]]. $X = A^{-1}B$ = [[1·12+(−2)·5],[(−1)·12+3·5]] = [[12−10],[−12+15]] = [[2],[3]]. So x = 2, y = 3, and x + y = 5. Verify: 3(2)+2(3) = 6+6 = 12 ✓; 2+3 = 5 ✓.

★ TOPPER TEMPLATE — 3 marks: 'Express A = [[2,4],[6,8]] as the sum of a symmetric and a skew-symmetric matrix.'

Most years

Step 1 [1 mark]	**State the decomposition formula**	Any square matrix A can be written as A = P + Q, where P = (1/2)(A + A') is symmetric and Q = (1/2)(A – A') is skew-symmetric. First write the transpose: A' = [[2,6],[4,8]].
Step 2 [1 mark]	**Build the symmetric part**	P = (1/2)(A + A') = (1/2)[[2+2, 4+6],[6+4, 8+8]] = (1/2)[[4,10],[10,16]] = [[2,5],[5,8]]. Check: P' = P ✓ (the off-diagonal 5's are equal), so P is symmetric.
Step 3 [1 mark]	**Build the skew part and verify the sum**	Q = (1/2)(A – A') = (1/2)[[2–2, 4–6],[6–4, 8–8]] = (1/2)[[0,–2],[2,0]] = [[0,–1],[1,0]]. Diagonal is all zero ✓ (skew condition). Verify: P + Q = [[2,5],[5,8]] + [[0,–1],[1,0]] = [[2,4],[6,8]] = A ✓.

COMMON LOSS OF MARKS:

- Forgetting the factor of 1/2 — writing P = A + A' instead of (1/2)(A + A').
- Writing the skew-symmetric matrix with a non-zero diagonal — by definition the diagonal must be all zeros.
- Not verifying P + Q = A at the end; the verification line is worth a mark and catches arithmetic slips.

★ TOPPER TEMPLATE — 3 marks: 'Find the inverse of A = [[2,3],[1,4]] using the adjoint method.'

Annual

Step 1 [1 mark]	**Compute the determinant and confirm it is non-zero**	|A| = (2)(4) – (3)(1) = 8 – 3 = 5. Since |A| = 5 ≠ 0, A is non-singular, so A^{-1} exists.
Step 2 [1 mark]	**Write the adjoint (2x2 shortcut)**	For a 2x2 matrix [[a,b],[c,d]], adj(A) = [[d,–b],[–c,a]] — swap the diagonal entries, negate the off-diagonal entries. So adj(A) = [[4,–3],[–1,2]].
Step 3 [1 mark]	**Assemble the inverse and verify**	A^{-1} = (1/|A|)·adj(A) = (1/5)[[4,–3],[–1,2]] = [[4/5, –3/5],[–1/5, 2/5]]. Verify: A·A^{-1} = (1/5)[[2,3],[1,4]]·[[4,–3],[–1,2]] = (1/5)[[8–3, –6+6],[4–4, –3+8]] = (1/5)[[5,0],[0,5]] = I ✓.

COMMON LOSS OF MARKS:

- Attempting the inverse without first checking |A| ≠ 0.
- Forgetting to divide by |A| — leaving the answer as adj(A) only.
- Sign error in the 2x2 adjoint shortcut: only the OFF-diagonal entries are negated, the diagonal entries are SWAPPED (not negated).

★ TOPPER TEMPLATE — 5 marks: 'Solve by the matrix method: 2x + 3y = 13 and x + y = 5.'

Annual

Step 1 [1 mark]	**Write the system as AX = B**	A = [[2,3],[1,1]], X = [[x],[y]], B = [[13],[5]]. The system AX = B has solution X = A^{-1}B provided |A| ≠ 0.
Step 2 [1 mark]	**Compute the determinant**	|A| = (2)(1) – (3)(1) = 2 – 3 = –1. Since |A| = –1 ≠ 0, the system has a unique solution and A^{-1} exists.
Step 3 [1 mark]	**Find the inverse**	adj(A) = [[1,–3],[–1,2]]. A^{-1} = (1/|A|)·adj(A) = (1/–1)[[1,–3],[–1,2]] = [[–1,3],[1,–2]].
Step 4 [1 mark]	**Multiply X = A^{-1}B**	X = A^{-1}B = [[–1,3],[1,–2]]·[[13],[5]] = [[(–1)(13)+(3)(5)],[(1)(13)+(–2)(5)]] = [[–13+15],[13–10]] = [[2],[3]]. So x = 2, y = 3.
Step 5 [1 mark]	**Verify in both original equations**	Check eq.1: 2(2) + 3(3) = 4 + 9 = 13 ✓. Check eq.2: 2 + 3 = 5 ✓. Both equations are satisfied, so x = 2, y = 3 is the correct solution.

COMMON LOSS OF MARKS:

- Not stating |A| ≠ 0 before claiming a unique solution exists.
- Sign slip when dividing the adjoint by a NEGATIVE determinant.
- Skipping the final verification — substituting back into both equations is worth a mark and proves the answer.

MARKING SCHEME — GENERAL NOTES

- For equality of matrices, state the corresponding-entry equations explicitly before solving.
- Symmetric/skew decomposition: ALWAYS include the 1/2 factor and verify P + Q = A; confirm the skew diagonal is all zeros.
- Matrix multiplication: show the row-times-column products; never assume AB = BA.
- 3×3 determinant: write the + – + sign pattern and keep each minor's own sign explicit.
- Inverse / system: compute |A| FIRST; if |A| = 0 state that the inverse does not exist. Always verify A·A^{-1} = I, or substitute the solution back into every equation.