

EXAM-DAY · 90-MIN REVISION CARD

Algebra (Matrices and Determinants)

Print this · Fold it · Carry to the exam-hall gate · Revise once · Then walk in.

FORMULAS & KEY RESULTS

ORDER: an $m \times n$ matrix has m rows, n columns. **Equality:** same order AND each corresponding entry equal.

TRANSPOSE: A' (or A^T) swaps rows and columns. $(A')' = A$; $(A+B)' = A'+B'$; $(kA)' = kA'$; $(AB)' = B'A'$ (REVERSED).

SYMMETRIC: $A' = A$. **SKEW-SYMMETRIC:** $A' = -A$ (every diagonal entry = 0).

DECOMPOSITION: $A = (1/2)(A+A') + (1/2)(A-A')$ = symmetric part + skew-symmetric part.

MATRIX MULTIPLY: $(m \times n) \cdot (n \times p) = (m \times p)$; inner dims must match. NOT commutative: $AB \neq BA$ in general.

IDENTITY: $A \cdot I = I \cdot A = A$. **ZERO product:** $AB = 0$ does NOT imply $A = 0$ or $B = 0$.

DET 2x2: $|\begin{bmatrix} a & b \\ c & d \end{bmatrix}| = ad - bc$.

DET 3x3 (expand along row 1): $a(ei - fh) - b(di - fg) + c(dh - eg)$ for $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$.

COFACTOR: $C(i,j) = (-1)^{i+j} \cdot M(i,j)$, M = minor. Sign grid: $+$ $-$ $+$ / $-$ $+$ $-$ / $+$ $-$ $+$.

ADJOINT: $\text{adj}(A)$ = transpose of the cofactor matrix. **INVERSE:** $A^{-1} = (1/|A|) \cdot \text{adj}(A)$, needs $|A| \neq 0$.

2x2 ADJOINT SHORTCUT: $\text{adj}[\begin{bmatrix} a & b \\ c & d \end{bmatrix}] = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (swap diagonal, negate off-diagonal).

SOLVE SYSTEM: $AX = B \Rightarrow X = A^{-1}B$ (if $|A| \neq 0$). **CRAMER:** $x = Dx/D$, $y = Dy/D$, $z = Dz/D$ where $D = |A|$.

TOP 5 PYQ PATTERNS

1 Type / order / equality of matrices (find x, y)

2 marks · 90% of years

State order as rows \times columns · for equality set corresponding entries equal · solve the resulting tiny linear system.

2 Symmetric + skew-symmetric decomposition

3 marks · 80% of years

Write A' · form $P = (1/2)(A+A')$ symmetric and $Q = (1/2)(A-A')$ skew · verify $P+Q=A$ and diagonal of Q is zero.

3 Determinant 3x3 / minors and cofactors

3 marks · 85% of years

Draw the sign grid first · expand along the row / column with most zeros · keep the alternating signs explicit.

4 Inverse via adjoint

4 marks · 85% of years

Compute $|A|$ first (stop if zero) · build cofactor matrix · transpose for adj · divide by $|A|$ · verify $A \cdot A^{-1} = I$.

5 Solve a linear system (matrix method / Cramer)

5 marks · 95% of years

Write $AX=B$ · confirm $|A| \neq 0$ · $X=A^{-1}B$ (or Cramer ratios) · substitute back into BOTH/ALL equations to verify.

90-MIN REVISION FLOW

0-15 min

Memorise the five backbone formulas — transpose rules, symmetric/skew test, 2x2 and 3x3 determinant, adjoint \rightarrow inverse. These five carry the whole chapter.

15-30 min

Drill 4 matrix-multiplication problems. For each, write the order check (inner dims match) BEFORE multiplying, and confirm $AB \neq BA$ on one example.

30-50 min

Do 3 symmetric/skew decompositions and 3 determinant evaluations. Draw the cofactor sign grid every single time on the 3x3.

50-70 min

Compute the inverse of two 2x2 and one 3x3 matrix via adjoint. Always check $|A| \neq 0$ first and verify $A \cdot A^{-1} = I$ at the end.

70-88 min

Solve two full linear systems by the matrix method, substituting back into every equation. This is the guaranteed 5-mark long answer.

88-90 min

Self-check — can you state, without looking, why a singular matrix has no inverse and why every skew-symmetric diagonal entry is zero? If yes, you are chapter-ready.

Confidence, not anxiety. You've practised this all year. Trust your steps. Don't change strategy on exam morning.
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