CHAPTER 2

Algebra (Matrices and Determinants)

CBSE Class 12 Applied Mathematics · Unit 2

CBSE · Applied Mathematics · Class 12

WHAT THIS CHAPTER DOES

A — Classify matrices by type and order, and test the rule for when a product AB is defined.

B — Add, scalar-multiply, and multiply matrices fluently — and prove to yourself that AB ≠ BA.

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's mission

1. Classify matrices by type and order, and test the rule for when a product AB is defined.
2. Add, scalar-multiply, and multiply matrices fluently — and prove to yourself that $AB \neq BA$.
3. Evaluate 2×2 and 3×3 determinants with the correct cofactor sign pattern.
4. Find the inverse of a matrix via adjoint, and use it to solve a system of linear equations end-to-end.

WHY THIS MATTERS

Why this chapter matters

1. The single highest-weight chapter in Class 12 Applied Maths — 8-12 marks every board paper, including a guaranteed 5-mark solve-the-system long answer.

2. Every later business model — input-output analysis, Markov chains, market equilibrium — reduces to a matrix equation $AX = B$ solved by exactly the methods learnt here.

3. It is procedural, not abstract: students who practise the steps cleanly and check their signs collect almost full marks. This is the highest marks-per-hour chapter in the book.

TOPIC

A

Matrices — types and order

THEOREM · LOAD-BEARING RESULT

Order, equality, and the matrix zoo

> " A matrix is a rectangular array of numbers arranged in rows and columns; an m×n matrix has m rows and n columns. Two matrices are equal only if they have the SAME order and every corresponding entry is equal.

STATEMENT

Types: ROW (1×n), COLUMN (m×1), SQUARE (n×n), DIAGONAL (square, all off-diagonal entries 0), SCALAR (diagonal with equal diagonal entries), IDENTITY I (diagonal with all diagonal entries 1),

WHY THIS MATTERS

- Order is the gatekeeper for every later operation — addition needs equal orders, multiplication needs matching inner dimensions
- Equality of matrices is itself a 1-2 mark PYQ that reduces to solving a tiny linear system, so it rewards a precise reading of the definition.

WATCH OUT FOR

NOTE Do NOT confuse a scalar matrix (equal non-zero diagonal entries) with the identity matrix (diagonal entries exactly 1). Every identity matrix is scalar, but not every scalar matrix is the identity.

WORKED EXAMPLE

Worked example — equality of matrices

1. QUESTION: Find x and y if [[2x, 5],[7, y–3]] = [[8, 5],[7, 4]].
2. STEP 1 — Both matrices are 2×2, so equality is possible. Equate corresponding entries: $2x = 8$ and $y - 3 = 4$. (The 5 = 5 and 7 = 7 entries are already consistent.)
3. STEP 2 — Solve each tiny equation: from $2x = 8$, $x = 4$; from $y - 3 = 4$, $y = 7$.
4. STEP 3 — State the answer: $x = 4$, $y = 7$. Substitute back: [[8, 5],[7, 4]] matches the right-hand side ✓.
5. TAKEAWAY: Equality of matrices is just a bundle of simultaneous equations, one per corresponding entry — read the definition literally and the question solves itself.

TOPIC

B

Algebra of matrices

THEOREM · LOAD-BEARING RESULT

Addition, scalar multiplication, and the multiplication rule

> Matrices of the same order are added entry-by-entry. A scalar k multiplies every entry. The product AB is defined only when the number of columns of A equals the number of rows of B, and matrix multiplication is NOT commutative.

STATEMENT

Addition: (A + B)(i,j) = a(i,j) + b(i,j), requires same order. Scalar: (kA)(i,j) = k·a(i,j). Multiplication: if A is m×n and B is n×p, then AB is m×p, with (AB)(i,j) = sum over k of a(i,k)·b(k,j) — the i-th row of A

WHY THIS MATTERS

- Matrix multiplication is the engine behind every application in the course — combining a quantity vector with a price matrix to get total cost, or applying a transition matrix in a Markov chain
- The row-times-column rule is mechanical once the dimension check becomes automatic.

WATCH OUT FOR

NOTE ALWAYS check the inner dimensions match before multiplying, and NEVER swap the order of factors: AB and BA are different products (and one may not even exist). Treating matrices like ordinary numbers is the most common source of lost marks here.

WORKED EXAMPLE

Worked example — multiplication and non-commutativity

1. QUESTION: With A = [[1,2],[3,4]] and B = [[5,6],[7,8]], compute AB and BA and show they differ.
2. STEP 1 — Dimension check: A is 2×2 and B is 2×2, inner dims (2 = 2) match, so both products exist and are 2×2.
3. STEP 2 — AB: row1·col1 = 1·5 + 2·7 = 19; row1·col2 = 1·6 + 2·8 = 22; row2·col1 = 3·5 + 4·7 = 43; row2·col2 = 3·6 + 4·8 = 50. So AB = [[19,22],[43,50]].
4. STEP 3 — BA: row1·col1 = 5·1 + 6·3 = 23; row1·col2 = 5·2 + 6·4 = 34; row2·col1 = 7·1 + 8·3 = 31; row2·col2 = 7·2 + 8·4 = 46. So BA = [[23,34],[31,46]].
5. CONCLUSION: AB = [[19,22],[43,50]] ≠ [[23,34],[31,46]] = BA. Matrix multiplication is NOT commutative — order matters every single time.

TOPIC

C

Transpose, symmetric, skew-symmetric

TOPIC

Transpose and the symmetric / skew-symmetric pair

TRANSPOSE — SWAP ROWS AND COLUMNS

The transpose of a matrix A, written A' or A^T, is obtained by turning every row into a column: the entry at row i, column j moves to row j, column i. If A is m×n then A' is n×m. The key algebraic rules are (A')' = A, (A + B)' = A' + B', (kA)' = kA', and crucially (AB)' = B'A' — the transpose of a

SYMMETRIC MATRIX — A' = A

A square matrix is symmetric if it equals its own transpose, A' = A, which means the entry at (i,j) equals the entry at (j,i) — the matrix is mirror-image across its main diagonal. For example [[2,5,1],[5,3,4],[1,4,7]] is symmetric because every off-diagonal pair matches.

SKEW-SYMMETRIC — A' = −A, ZERO DIAGONAL

A square matrix is skew-symmetric if its transpose is its negative, A' = −A, so the entry at (i,j) is the negative of the entry at (j,i). A direct consequence is that every main-diagonal entry must be ZERO: a(i,i) = −a(i,i) forces a(i,i) = 0. For example [[0,3,−2],[−3,0,5],[2,−5,0]] is skew-

EVERY SQUARE MATRIX = SYMMETRIC + SKEW

A powerful and frequently-tested result: ANY square matrix A can be written uniquely as the sum of a symmetric matrix and a skew-symmetric matrix, namely A = (1/2)(A + A') + (1/2)(A − A'). The first part P = (1/2)(A + A') is always symmetric, and the second part Q = (1/2)(A

TOPIC

D

Determinants, minors, cofactors

THEOREM · LOAD-BEARING RESULT

Determinants and the cofactor sign pattern

> The determinant is a single number associated with a SQUARE matrix. For a 2×2 it is ad – bc. For larger matrices it is computed by expanding along a row or column using minors and cofactors, where the cofactor carries an alternating (–1)^(i+j) sign.

STATEMENT

2×2: |[[a,b],[c,d]]| = ad – bc. MINOR M(i,j): the determinant left after deleting row i and column j. COFACTOR C(i,j) = (–1)^(i+j)·M(i,j). 3×3 expansion along row 1 for [[a,b,c],[d,e,f],[g,h,i]]: |A| = a(ei – fh) – b(di –

WHY THIS MATTERS

- The determinant decides whether a matrix is invertible (|A| ≠ 0) and whether a linear system has a unique solution
- It is the gateway to both the inverse and Cramer's rule, so a clean determinant is the foundation of the chapter's two biggest marks-earners.

WATCH OUT FOR

NOTE The alternating sign is the #1 trap. The middle term of a 3×3 row-1 expansion is SUBTRACTED: it is –b(...), not +b(...). Draw the + – + / – + – / + – + grid in the margin before you start and you will never drop a sign.

WORKED EXAMPLE

Worked example — 3×3 determinant

1. QUESTION: Evaluate $|A|$ for $A = [[1,2,3],[4,5,6],[7,8,10]]$.
2. STEP 1 — Expand along row 1 with the sign pattern + – +: $|A| = 1 \cdot M_{11} - 2 \cdot M_{12} + 3 \cdot M_{13}$.
3. STEP 2 — Minors: $M_{11} = |[[5,6],[8,10]]| = 5 \cdot 10 - 6 \cdot 8 = 50 - 48 = 2$. $M_{12} = |[[4,6],[7,10]]| = 4 \cdot 10 - 6 \cdot 7 = 40 - 42 = -2$. $M_{13} = |[[4,5],[7,8]]| = 4 \cdot 8 - 5 \cdot 7 = 32 - 35 = -3$.
4. STEP 3 — Combine with signs: $|A| = 1 \cdot (2) - 2 \cdot (-2) + 3 \cdot (-3) = 2 + 4 - 9 = -3$.
5. CHECK: The middle term $-2 \cdot (-2) = +4$ (subtracting a negative) — note how the sign pattern AND the minor's own sign both matter. Final answer: $|A| = -3$. Since $|A| \neq 0$, this matrix is invertible.

TOPIC

E

Adjoint, inverse, solving systems

THEOREM · LOAD-BEARING RESULT

Adjoint, inverse, and the matrix method

> "The adjoint of A is the transpose of its cofactor matrix. The inverse is $A^{-1} = (1/|A|)\cdot adj(A)$, which exists only when $|A| \neq 0$. A linear system $AX = B$ with $|A| \neq 0$ has the unique solution $X = A^{-1}B$.

STATEMENT

$adj(A) = [C(i,j)]^T$ — replace each entry by its cofactor, then transpose. $A^{-1} = (1/|A|)\cdot adj(A)$, defined iff $|A| \neq 0$ (A is then non-singular). For a 2×2, adj[[a,b],[c,d]] = [[d,–b],[–c,a]]. SOLVING: write AX = B;

WHY THIS MATTERS

- This is the climax of the chapter and the guaranteed 5-mark long answer
- Almost every board paper asks the student to solve a 2- or 3-variable linear system — often dressed as a small business problem (cost of commodities) — by exactly this matrix method.

WATCH OUT FOR

NOTE ALWAYS compute |A| FIRST. If |A| = 0 the inverse does not exist — stop and state so, never push on with the adjoint formula. And remember adj(A) is the transpose of the COFACTOR matrix, not the transpose of A itself.

WORKED EXAMPLE

Worked example — solve a system by the matrix method

1. QUESTION: Solve by the matrix method: $3x + 2y = 12$ and $x + y = 5$.

2. STEP 1 — Write AX = B: A = [[3,2],[1,1]], X = [[x],[y]], B = [[12],[5]].

3. STEP 2 — Determinant: $|A| = (3)(1) - (2)(1) = 3 - 2 = 1$. Since $|A| = 1 \neq 0$, a unique solution exists.

4. STEP 3 — Inverse: adj(A) = [[1,−2],[−1,3]]; $A^{-1} = (1/1)\cdot adj(A)$ = [[1,−2],[−1,3]].

5. STEP 4 — Multiply: $X = A^{-1}B$ = [[1,−2],[−1,3]]·[[12],[5]] = [[1·12 + (−2)·5],[(−1)·12 + 3·5]] = [[12−10],[−12+15]] = [[2],[3]]. So $x = 2$, $y = 3$.

6. STEP 5 — Verify: $3(2) + 2(3) = 6 + 6 = 12$ ✓ and $2 + 3 = 5$ ✓. Both equations hold, so $x = 2$, $y = 3$ is correct.

TOPIC

Matrix multiplication is commutative

TRAP → TRUTH

× **MISTAKE** AB = BA always, just like ordinary number multiplication.

✓ **CORRECT** Matrix multiplication is NOT commutative — in general $AB \neq BA$. They may even have different orders, so one product can exist while the other does not. Example: with A = [[1,2],[3,4]] and B = [[0,1],[1,0]], AB = [[2,1],[4,3]] but BA = [[3,4],[1,2]] — clearly different. Always preserve the order of factors; never swap them.

TOPIC

Order required for matrix multiplication

TRAP → TRUTH

× **MISTAKE** Any two matrices can be multiplied.

✓ **CORRECT** The product AB is defined ONLY when the number of COLUMNS of A equals the number of ROWS of B. An (m x n) matrix times an (n x p) matrix gives an (m x p) matrix — the inner dimensions must match and they cancel, the outer dimensions survive. If A is 2x3 and B is 2x3, AB is undefined. Check dimensions before multiplying.

TOPIC

Inverse exists for every matrix

TRAP → TRUTH

× **MISTAKE** Every square matrix has an inverse.

✓ **CORRECT** A square matrix A has an inverse if and only if its determinant is NON-ZERO ($|A| \neq 0$); such a matrix is called non-singular. If $|A| = 0$ the matrix is singular and A^{-1} does NOT exist. ALWAYS compute $|A|$ first — if it is zero, stop and state 'inverse does not exist' rather than attempting the adjoint formula.

TOPIC

Adjoint vs transpose confusion

TRAP → TRUTH

× **MISTAKE** The adjoint of A is just the transpose of A.

✓ **CORRECT** The adjoint of A is the TRANSPOSE of the matrix of COFACTORS of A — not the transpose of A itself. Steps: (1) replace each element by its cofactor (signed minor), (2) then transpose that cofactor matrix. $A^{-1} = (1/|A|) \times adj(A)$. Skipping the cofactor step and just transposing A is a complete-method error that loses all the marks.

TOPIC

Cofactor sign pattern

TRAP → TRUTH

× **MISTAKE** A cofactor equals its minor; the sign does not matter.

✓ **CORRECT** The cofactor $C(i,j) = (-1)^{(i+j)} \times M(i,j)$, where $M(i,j)$ is the minor. The sign follows the checkerboard pattern: + − + / − + − / + − +. Forgetting the $(-1)^{(i+j)}$ sign on the off-diagonal cofactors is the #1 cause of a wrong determinant or wrong adjoint. Write the sign grid in the margin before you start.

TOPIC

Skew-symmetric diagonal entries

TRAP → TRUTH

× **MISTAKE** A skew-symmetric matrix can have any numbers on its main diagonal.

✓ **CORRECT** For a skew-symmetric matrix, $A' = -A$, which forces every diagonal element $a(i,i) = -a(i,i)$, hence $2 \cdot a(i,i) = 0$, so EVERY main-diagonal entry must be ZERO. If a candidate writes a skew-symmetric matrix with a non-zero diagonal entry, it is wrong by definition.

TOPIC

Order of (AB)' transpose of a product

TRAP → TRUTH

× **MISTAKE** (AB)' = A'B' — the transpose distributes in the same order.

✓ **CORRECT** The transpose of a product REVERSES the order: (AB)' = B'A'. This 'reversal' rule mirrors the inverse rule $(AB)^{-1} = B^{-1}A^{-1}$. Writing A'B' is a standard 1-mark trap that examiners deliberately set.

TOPPER TEMPLATE · MARK-BY-MARK

3 marks: 'Express A = [[2,4],[6,8]] as the sum of a symmetric and a skew-symmetric matrix.'

1 STATE THE DECOMPOSITION FORMULA (1 m)

Any square matrix A can be written as A = P + Q, where P = (1/2)(A + A') is symmetric and Q = (1/2)(A – A') is skew-symmetric. First write the transpose: A' = [[2,6],[4,8]].

2 BUILD THE SYMMETRIC PART (1 m)

P = (1/2)(A + A') = (1/2)[[2+2, 4+6],[6+4, 8+8]] = (1/2)[[4,10],[10,16]] = [[2,5],[5,8]]. Check: P' = P ✓ (the off-diagonal 5's are equal), so P is symmetric.

3 BUILD THE SKEW PART AND VERIFY THE SUM (1 m)

Q = (1/2)(A – A') = (1/2)[[2–2, 4–6],[6–4, 8–8]] = (1/2)[[0,–2],[2,0]] = [[0,–1],[1,0]]. Diagonal is all zero ✓ (skew condition). Verify: P + Q = [[2,5],[5,8]] + [[0,–1],[1,0]] = [[2,4],[6,8]] = A ✓.

TOPPER TEMPLATE · MARK-BY-MARK

3 marks: 'Find the inverse of A = [[2,3],[1,4]] using the adjoint method.'

1 COMPUTE THE DETERMINANT AND CONFIRM IT IS NON-ZERO (1 m)

$|A| = (2)(4) - (3)(1) = 8 - 3 = 5$. Since $|A| = 5 \neq 0$, A is non-singular, so A^{-1} exists.

2 WRITE THE ADJOINT (2X2 SHORTCUT) (1 m)

For a 2x2 matrix [[a,b],[c,d]], adj(A) = [[d,–b],[–c,a]] — swap the diagonal entries, negate the off-diagonal entries. So adj(A) = [[4,–3],[–1,2]].

3 ASSEMBLE THE INVERSE AND VERIFY (1 m)

$A^{-1} = (1/|A|)\cdot adj(A)$ = (1/5)[[4,–3],[–1,2]] = [[4/5, –3/5],[–1/5, 2/5]]. Verify: $A\cdot A^{-1}$ = (1/5)[[2,3],[1,4]]·[[4,–3],[–1,2]] = (1/5)[[8–3, –6+6],[4–4, –3+8]] = (1/5)[[5,0],[0,5]] = I ✓.

TOPPER TEMPLATE · MARK-BY-MARK

5 marks: 'Solve by the matrix method: $2x + 3y = 13$ and $x + y = 5$.'

1 WRITE THE SYSTEM AS AX = B (1 m)

$A = [[2,3],[1,1]]$, $X = [[x],[y]]$, $B = [[13],[5]]$. The system $AX = B$ has solution $X = A^{-1}B$ provided $|A| \neq 0$.

2 COMPUTE THE DETERMINANT (1 m)

$|A| = (2)(1) - (3)(1) = 2 - 3 = -1$. Since $|A| = -1 \neq 0$, the system has a unique solution and A^{-1} exists.

3 FIND THE INVERSE (1 m)

$\text{adj}(A) = [[1,-3],[-1,2]]$. $A^{-1} = (1/|A|)\cdot\text{adj}(A) = (1/-1)[[1,-3],[-1,2]] = [[-1,3],[1,-2]]$.

4 MULTIPLY X = A⁻¹B (1 m)

$X = A^{-1}B = [[-1,3],[1,-2]]\cdot[[13],[5]] = [[(-1)(13)+(3)(5)],[(1)(13)+(-2)(5)]] = [[-13+15],[13-10]] = [[2],[3]]$. So $x = 2$, $y = 3$.

5 VERIFY IN BOTH ORIGINAL EQUATIONS (1 m)

Check eq.1: $2(2) + 3(3) = 4 + 9 = 13$ ✓. Check eq.2: $2 + 3 = 5$ ✓. Both equations are satisfied, so $x = 2$, $y = 3$ is the correct solution.

PYQ PATTERNS

Top PYQ patterns to drill

#1	Given a matrix, state its order / identify its type, or find x, y from an equality of matrices. (1-2 marks)	Annual
#2	Express a given square matrix as the sum of a symmetric and a skew-symmetric matrix, or verify (A+A')/2 is symmetric. (2-3 marks)	Most years
#3	Compute a product AB (and show AB ≠ BA), or evaluate a 3x3 determinant. (3 marks)	Annual
#4	Find the inverse of a 2x2 or 3x3 matrix using adjoint, given det ≠ 0. (3-4 marks)	Annual
#5	Solve a system of two or three linear equations by the matrix (inverse) method or Cramer's rule. (5 marks)	Annual

MARKS DISTRIBUTION

10-year marks distribution

10-YEAR PYQ MARKS DISTRIBUTION

Types of matrices · order · equality	10%
Transpose · symmetric · skew-symmetric	14%
Algebra of matrices (add · scalar · multiply)	18%
Determinants · minors · cofactors	16%
Adjoint and inverse of a matrix	16%
Solving linear systems (matrix method · Cramer)	26%

RECAP · MEMORISE THESE

Recap — the five backbone results

1 Types & order — m×n = m rows, n columns. Equality needs same order AND equal corresponding entries. Product AB defined iff cols(A) = rows(B).

2 Algebra of matrices — Add entry-by-entry (same order). (AB)(i,j) = row i of A · column j of B. AB ≠ BA in general; AB = 0 does not force A or B = 0.

3 Transpose & symmetry — (AB)' = B'A' (reversed). Symmetric: A' = A. Skew: A' = –A with zero diagonal. A = (1/2)(A+A') + (1/2)(A–A').

4 Determinants — 2×2: ad – bc. 3×3: expand with + – + signs and

5 Inverse & solving — A^{-1} = (1/|A|)·adj(A), needs |A| ≠ 0. System

WHAT'S NEXT

What's next

→ Chapter 3 — Calculus (applications of derivatives / marginal analysis).

→ Sit the 15-MCQ Quick Drill for this chapter.

→ Then the full Board-Pattern Paper — 30 marks.

You've mastered Unit 2 — the heaviest chapter.

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