ANSWER KEY & MARKING SCHEME · CBSE CLASS 12

Calculus — Differentiation, Applications & Integration

Applied Mathematics · Chapter 3 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Short calculation (2 × 3 = 6 marks)

Q1. For $f(x) = 2x^3 - 9x^2 + 12x + 1$, find f'(x) and f''(x). [2 marks]

Ans: $f'(x) = 6x^2 - 18x + 12$; $f''(x) = 12x - 18$. (Refer to chapter notes deck for the differentiation steps.)

Q2. Evaluate the definite integral ∫ from 1 to 3 of (2x + 1) dx. [2 marks]

Ans: $\int(2x+1)dx = x^2 + x$. Evaluate 1→3: $(9+3) - (1+1) = 12 - 2 = 10$. (See notes deck.)

Q3. If $C(x) = 3x^2 + 5x + 40$ is the total cost, find the marginal cost when x = 8 and interpret it. [2 marks]

Ans: MC = dC/dx = 6x + 5; at x = 8, MC = 48 + 5 = ₹53 — the approximate cost of the 9th unit. (See topper template.)

Section B — Application problem (3 × 4 = 12 marks)

Q4. The total cost of producing x units is $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when x = 50 and state its meaning. [3 marks]

Ans: Full mark-by-mark model answer is in topper_answer_templates (appmaths_marginal_3mark). MC = $0.015x^2 - 0.04x + 30$; at x = 50, MC = ₹65.50 — cost of the 51st unit.

Q5. Find the intervals on which $f(x) = x^3 - 6x^2 + 9x + 2$ is increasing and on which it is decreasing. [3 marks]

Ans: $f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$. f' > 0 (increasing) for x < 1 or x > 3; f' < 0 (decreasing) for 1 < x < 3. (See chapter notes.)

Q6. Evaluate ∫ from 0 to 2 of $4x(x^2 + 1)$ dx by expanding or by substitution. [3 marks]

Ans: Expand: $4x^3 + 4x$; $\int = x^4 + 2x^2$. Evaluate 0→2: (16 + 8) − 0 = 24. (Substitution $u=x^2+1$ gives the same.) (See notes deck.)

Q7. The demand function is p = 40 − x and the market price is ₹20. Find the consumer surplus. [3 marks]

Ans: Equilibrium: $20 = 40 - x \Rightarrow x_0 = 20$. CS = $\int_0^{20}[(40-x)-20]dx = \int_0^{20}(20-x)dx = [20x - x^2/2]_0^{20} = 400 - 200$ = ₹200. (See topper template.)

Section C — Long-answer (6 × 2 = 12 marks)

Q8. A firm's revenue function is $R(x) = 60x - x^2$ and its cost function is $C(x) = x^2 + 20x + 100$ (in ₹, x = number of units). Find the level of output that maximises profit, verify it is a maximum using the second-derivative test, and find the maximum profit. [6 marks]

Ans: $P(x) = R - C = (60x - x^2) - (x^2 + 20x + 100) = -2x^2 + 40x - 100$. $P'(x) = -4x + 40 = 0 \Rightarrow x = 10$. $P''(x) = -4 < 0 \Rightarrow$ MAXIMUM. Maximum profit P(10) = −2(100) + 400 − 100 = ₹100. Check (MR = MC): R'(x) = 60 − 2x, C'(x) = 2x + 20; 60 − 2x = 2x + 20 ⇒ x = 10. ✓ Output = 10 units, maximum profit = ₹100.

Q9. The total cost of producing x units is $C(x) = x^2 + 64x + 100$ (in ₹). Find the output at which the AVERAGE cost per unit is minimum, verify with the second-derivative test, and find the minimum average cost. [6 marks]

Ans: Average cost $A(x) = C(x)/x = x + 64 + 100/x = x + 64 + 100x^{-1}$. $A'(x) = 1 - 100/x^2 = 0 \Rightarrow x^2 = 100 \Rightarrow x = 10$ (reject x = −10). $A''(x) = 200/x^3$; A''(10) = 200/1000 = 0.2 > 0 ⇒ MINIMUM. Minimum average cost A(10) = 10 + 64 + 100/10 = 10 + 64 + 10 = ₹84 per unit. Output = 10 units, minimum average cost = ₹84/unit.

★ TOPPER TEMPLATE — 3 marks: 'The total cost of producing x units is $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Find the marginal cost when x = 50 and interpret it.'

Annual

Step 1 [1 mark]	**Differentiate the cost function**	Marginal cost is MC = dC/dx. Differentiate term by term: $d/dx(0.005x^3) = 0.015x^2$; $d/dx(-0.02x^2) = -0.04x$; $d/dx(30x) = 30$; $d/dx(5000) = 0$. Hence $MC = 0.015x^2 - 0.04x + 30$.
Step 2 [1 mark]	**Substitute x = 50**	At x = 50: $MC = 0.015(50^2) - 0.04(50) + 30 = 0.015(2500) - 2 + 30 = 37.5 - 2 + 30 =$ ₹65.50.
Step 3 [1 mark]	**Interpret in words**	Interpretation: when 50 units are already being produced, the approximate cost of producing the 51st (one more) unit is ₹65.50. Marginal cost is the per-unit cost AT THE MARGIN, not the average cost over all 50 units.

COMMON LOSS OF MARKS:

- Computing C(50)/50 (average cost) instead of dC/dx (marginal cost).
- Differentiation slip on the $0.005x^3$ term — d/dx gives $0.015x^2$, not 0.005·3 mis-multiplied.
- Omitting the interpretation sentence — the third mark is reserved for the economic meaning.

★ TOPPER TEMPLATE — 5 marks: 'A firm's revenue is $R(x) = 60x - x^2$ and cost is $C(x) = x^2 + 20x + 100$ (₹, x = units). Find the output that maximises profit, and the maximum profit.'

Annual

Step 1 [1 mark]	**Write the profit function**	Profit $P(x) = R(x) - C(x) = (60x - x^2) - (x^2 + 20x + 100) = 60x - x^2 - x^2 - 20x - 100 = -2x^2 + 40x - 100$.
Step 2 [1 mark]	**Differentiate and set to zero**	$P'(x) = -4x + 40$. Set $P'(x) = 0$: $-4x + 40 = 0 \Rightarrow 4x = 40 \Rightarrow x = 10$. So x = 10 is the only critical point.
Step 3 [1 mark]	**Apply the second-derivative test**	$P''(x) = -4 < 0$ for all x. Since $P''(10) = -4 < 0$, the critical point x = 10 is a LOCAL MAXIMUM. (Equivalently MR = MC: $R'(x) = 60 - 2x$, $C'(x) = 2x + 20$; setting equal: $60 - 2x = 2x + 20 \Rightarrow 40 = 4x \Rightarrow x = 10$. ✓)
Step 4 [1 mark]	**Compute the maximum profit**	Maximum profit $= P(10) = -2(10^2) + 40(10) - 100 = -2(100) + 400 - 100 = -200 + 400 - 100 =$ ₹100.
Step 5 [1 mark]	**State the conclusion**	Therefore the firm should produce 10 units to maximise profit, and the maximum profit is ₹100. Check via MR = MC confirmed x = 10, so the answer is consistent by two independent methods.

COMMON LOSS OF MARKS:

- Maximising revenue R(x) instead of profit P(x) = R – C.
- Skipping the second-derivative test — examiners require explicit confirmation that the critical point is a maximum, not a minimum.
- Arithmetic slip in P(10): note $-2(100) = -200$, then $-200 + 400 - 100 = 100$. Show the substitution fully.

★ TOPPER TEMPLATE — 4 marks: 'The demand function is $p = 50 - 2x$ and the market (equilibrium) price is $p_0 = 30$. Find the consumer surplus.'

Most years (2022 onwards)

Step 1 [1 mark]	**Find the equilibrium quantity**	At the market price $p_0 = 30$, the demand curve gives the quantity sold: $30 = 50 - 2x \Rightarrow 2x = 20 \Rightarrow x_0 = 10$ units. So at equilibrium, quantity $x_0 = 10$ and price $p_0 = 30$.
Step 2 [1 mark]	**Write the consumer-surplus integral**	Consumer surplus CS = ∫ from 0 to x_0 of [demand(x) – p_0] dx = ∫ from 0 to 10 of [(50 – 2x) – 30] dx = ∫ from 0 to 10 of (20 – 2x) dx.
Step 3 [1 mark]	**Integrate**	$\int (20 - 2x)\,dx = 20x - x^2$. Evaluate from 0 to 10: $[20(10) - 10^2] - [20(0) - 0^2] = (200 - 100) - 0 = 100$.
Step 4 [1 mark]	**State the answer with units**	Consumer surplus = ₹100. This is the total extra value the consumers gained — the difference between what they were willing to pay (the area under the demand curve up to 10 units) and what they actually paid (price 30 × 10 units = 300).

COMMON LOSS OF MARKS:

- Integrating the demand function alone without subtracting p_0 — that gives the area under the curve, not the surplus.
- Forgetting to find the equilibrium quantity x_0 first (you cannot set the upper limit without it).
- Dropping units (₹) on the final surplus.

MARKING SCHEME — GENERAL NOTES

- On differentiation, apply the power rule term by term: $d/dx(x^n) = n \cdot x^{n-1}$; constants differentiate to 0.
- On maxima-minima, ALWAYS solve $f'(x) = 0$ AND confirm with the second-derivative test ($f'' < 0$ max, $f'' > 0$ min) — the test is worth a mark.
- Distinguish MARGINAL cost MC = dC/dx from AVERAGE cost C(x)/x — they are different functions.

- Indefinite integrals MUST carry + C; definite integrals evaluate to a NUMBER with no + C.
- On consumer/producer surplus, find the equilibrium quantity x_0 FIRST, then subtract the price line before integrating; state the answer in ₹.