

EXAM-DAY · 90-MIN REVISION CARD

# Calculus – Differentiation, Applications & Integration

Print this · Fold it · Carry to the exam-hall gate · Revise once · Then walk in.

## FORMULAS & KEY RESULTS

POWER RULE:  $d/dx(x^n) = n \cdot x^{n-1}$ ;  
 $\int x^n dx = x^{n+1}/(n+1) + C$  ( $n \neq -1$ )

SECOND DERIVATIVE:  $f''(x) = d/dx[f'(x)]$  — rate of change of the slope; concavity

INCREASING/DECREASING:  $f'(x) > 0 \Rightarrow$  increasing;  $f'(x) < 0 \Rightarrow$  decreasing

MAXIMA/MINIMA: solve  $f'(x) = 0$  for critical  $x$ ; then  $f''(x) < 0 \Rightarrow$  MAX,  $f''(x) > 0 \Rightarrow$  MIN

MARGINAL COST:  $MC = dC/dx$  (cost of one more unit);  
AVERAGE COST =  $C(x)/x$  (different!)

MARGINAL REVENUE:  $MR = dR/dx$ ; PROFIT MAX when  $MR = MC$  and  $P''(x) < 0$

PROFIT:  $P(x) = R(x) - C(x)$ ;  
REVENUE:  $R(x) = \text{price} \times \text{quantity} = p \cdot x$

INDEFINITE INTEGRAL: always add  $+ C$ ; DEFINITE INTEGRAL:  
 $\int_a^b = F(b) - F(a)$  ( $a$  number, no  $C$ )

SUBSTITUTION (definite): change the LIMITS to the new variable, or back-substitute before evaluating

CONSUMER SURPLUS:  $CS = \int_{p_0}^{x_0} [D(x) - p_0] dx = \int_{p_0}^{x_0} D(x) dx - p_0 \cdot x_0$

PRODUCER SURPLUS:  $PS = \int_{p_0}^{x_0} [p_0 - S(x)] dx = p_0 \cdot x_0 - \int_{p_0}^{x_0} S(x) dx$

DIFFERENTIAL EQUATION: ORDER = highest derivative's order; DEGREE = power of that highest derivative (when polynomial in derivatives)

## TOP 5 PYQ PATTERNS

### 1 Marginal cost / marginal revenue at a given output

3 marks · 90% of years

Differentiate  $C(x)$  or  $R(x)$  · substitute the given  $x$  · interpret as 'cost/revenue of one more unit'.

### 2 Profit / revenue maximisation (maxima-minima)

5 marks · 95% of years

Form  $P(x) = R - C$  · solve  $P'(x) = 0$  · confirm with  $P''(x) < 0$  · compute  $P$  at that  $x$  · state the optimal output.

### 3 Increasing / decreasing intervals

3 marks · 70% of years

Find  $f'(x)$  · solve  $f'(x) = 0$  for boundary points · test the sign of  $f'$  in each interval · state increasing where  $f' > 0$ , decreasing where  $f' < 0$ .

### 4 Definite integral (often by substitution)

3 marks · 85% of years

Choose substitution  $u=g(x)$  · change the limits · integrate · evaluate  $F(\text{upper}) - F(\text{lower})$ . No  $+C$  on a definite integral.

### 5 Consumer / producer surplus

4 marks · 75% of years

Find equilibrium  $x_0$  from price ·  $CS = \int_{p_0}^{x_0} [D(x) - p_0] dx$  (or  $PS = \int_{p_0}^{x_0} [p_0 - S(x)] dx$ ) · integrate · state with ₹ units.

## 90-MIN REVISION FLOW

### 0-15 min

Memorise the spine: power rule (differentiate + integrate), the second-derivative test rule ( $f'' < 0 \Rightarrow$  max,  $f'' > 0 \Rightarrow$  min),  $MC = dC/dx$ ,  $MR = dR/dx$ , and the consumer/producer surplus integrals.

### 15-35 min

Drill 3 marginal cost/revenue problems and 2 increasing/decreasing problems. For each, differentiate cleanly and substitute carefully.

### 35-60 min

Solve 2 full profit-maximisation long-answers end-to-end: form  $P(x)$ , solve  $P'=0$ , confirm with  $P'' < 0$ , compute the maximum. This is the highest-yield block.

### 60-78 min

Do 3 definite integrals — at least one by substitution where you MUST change the limits. Then one consumer-surplus and one producer-surplus problem.

### 78-90 min

Self-check: state from memory the second-derivative test, the difference between MC and average cost, and the consumer-surplus formula. If all three are solid, you are chapter-ready.

**Confidence, not anxiety.** You've practised this all year. Trust your steps. Don't change strategy on exam morning.  
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