ANSWER KEY & MARKING SCHEME · CBSE CLASS 12

Probability Distributions

Applied Mathematics · Chapter 4 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — Short calculation (2 × 3 = 6 marks)

Q1. If $X \sim B(10, 0.5)$, find E(X) and Var(X). [2 marks]

Ans: $E(X) = np = 10 \times 0.5 = 5$; $Var(X) = np(1-p) = 10 \times 0.5 \times 0.5 = 2.5$. (See chapter notes deck.)

Q2. For $X \sim N(\mu = 100, \sigma^2 = 25)$, find the z-score corresponding to X = 110. [2 marks]

Ans: $\sigma = \sqrt{25} = 5$; $z = (110 - 100)/5 = 2.0$. (See notes deck and topper template for the standardisation procedure.)

Q3. $X \sim Po(\lambda = 2)$. Given $e^{-2} = 0.1353$, find P(X = 0) and P(X ⩾ 1). [2 marks]

Ans: $P(X = 0) = e^{-2} \cdot 2^0 / 0! = 0.1353$. $P(X \geqslant 1) = 1 - 0.1353 = 0.8647$. (See topper template.)

Section B — Application problem (3-4 marks each, 11 marks)

Q4. Two fair coins are tossed. Let X = number of heads. Construct the probability distribution of X and find E(X). [3 marks]

Ans: Full mark-by-mark model is in topper_answer_templates (appmaths_distrib_table_3mark). Table: X = 0,1,2 with P = 1/4, 1/2, 1/4; $\Sigma = 1$ ✓; $E(X) = 0(1/4) + 1(1/2) + 2(1/4) = 1$.

Q5. X has P(X = 1) = 0.2, P(X = 2) = 0.5, P(X = 3) = 0.3. Find E(X), Var(X) and σ. [4 marks]

Ans: $E(X) = 0.2 + 1.0 + 0.9 = 2.1$. $E(X^2) = 0.2 + 2.0 + 2.7 = 4.9$. $Var(X) = 4.9 - (2.1)^2 = 4.9 - 4.41 = 0.49$. $\sigma = \sqrt{0.49} = 0.7$. (Tabulate columns x, p, xp, x^2p — see notes deck.)

Q6. The number of defective bulbs per batch follows a Poisson distribution with mean $\lambda = 2$. Find P(X = 0) and P(X ⩾ 1). Use $e^{-2} = 0.1353$. [4 marks]

Ans: Full mark-by-mark model in topper_answer_templates (appmaths_poisson_4mark). $P(X = 0) = 0.1353$; $P(X \geqslant 1) = 1 - 0.1353 = 0.8647$.

Section C — Long-answer (5 + 6 = 11 marks)

Q7. The probability that a student passes a particular test is 0.6. Five students take the test independently. (a) Find P(exactly 3 pass). (b) Find the mean and variance of the number who pass. [5 marks]

Ans: Let X = number who pass, $X \sim B(n = 5, p = 0.6)$, q = 0.4. PMF: $P(X = k) = C(5, k)(0.6)^k(0.4)^{5-k}$. (a) $P(X = 3) = C(5,3) \cdot (0.6)^3 \cdot (0.4)^2 = 10 \cdot 0.216 \cdot 0.16 = 10 \cdot 0.03456 = 0.3456$. (b) Mean $E(X) = np = 5 \times 0.6 = 3$; Variance $Var(X) = np(1 - p) = 5 \times 0.6 \times 0.4 = 1.2$; $\sigma = \sqrt{1.2} \approx 1.095$. Conclusion: P(exactly 3 pass) = 0.3456; mean = 3; variance = 1.2.

Q8. The heights of adult males in a city are normally distributed with mean $\mu = 170$ cm and standard deviation $\sigma = 8$ cm. (a) Find the probability that a randomly chosen adult male is at most 178 cm tall. (b) Find the probability that he is taller than 186 cm. Use $\Phi(1.00) = 0.8413$ and $\Phi(2.00) = 0.9772$. [6 marks]

Ans: $X \sim N(\mu = 170, \sigma = 8)$. (a) Standardise X = 178: $z = (178 - 170)/8 = 1.00$. $P(X \leqslant 178) = P(Z \leqslant 1.00) = \Phi(1.00) = 0.8413$. So about 84.13% of adult males are 178 cm or shorter. (b) Standardise X = 186: $z = (186 - 170)/8 = 2.00$. $P(X \leqslant 186) = \Phi(2.00) = 0.9772$, so $P(X > 186) = 1 - 0.9772 = 0.0228$. About 2.28% of adult males are taller than 186 cm. Sanity check: $178 = \mu + \sigma \Rightarrow$ ~84% below (matches empirical rule); $186 = \mu + 2\sigma \Rightarrow$ ~97.7% below (matches empirical rule).

★ TOPPER TEMPLATE — 3 marks: 'Two fair coins are tossed. Let X be the number of heads. Construct the probability distribution of X and find E(X).'

Annual

Step 1 [1 mark]	**List sample space and values of X**	The sample space of two fair coin tosses is S = {HH, HT, TH, TT}, each with probability 1/4. X = number of heads can take values 0, 1, 2. X = 0 occurs only on TT (prob 1/4); X = 1 occurs on HT or TH (prob 2/4 = 1/2); X = 2 occurs only on HH (prob 1/4).
Step 2 [1 mark]	**Write the probability distribution table**	Distribution table: X = 0, 1, 2 with P(X) = 1/4, 1/2, 1/4. Check: 1/4 + 1/2 + 1/4 = 1 ✓ (sum of probabilities = 1).
Step 3 [1 mark]	**Compute E(X)**	$E(X) = \Sigma x_i \cdot p_i = 0\cdot(1/4) + 1\cdot(1/2) + 2\cdot(1/4) = 0 + 0.5 + 0.5 = 1$. So the expected number of heads in two tosses is 1.

COMMON LOSS OF MARKS:

- Writing P(X=1) = 1/4 instead of 1/2 (forgetting HT and TH are two outcomes, not one).
- Failing to verify $\Sigma p_i = 1$ — examiners explicitly reserve marks for that check.
- Computing the mean as (0 + 1 + 2)/3 = 1 by coincidence; the value happens to match but the METHOD is wrong and earns no marks.

★ TOPPER TEMPLATE — 5 marks: 'The probability that a student passes a test is 0.6. Five students take the test independently. Find (a) P(exactly 3 pass), (b) the mean and variance of the number who pass.'

Annual

Step 1 [1 mark]	**Identify the distribution**	Let X = number of students who pass out of n = 5. Each trial is independent with success probability p = 0.6 (and failure probability q = 1 − p = 0.4). So X ~ B(5, 0.6) — a binomial distribution. PMF: $P(X = k) = C(5, k)\,(0.6)^k\,(0.4)^{5-k}$.
Step 2 [2 marks]	**Compute P(X = 3)**	$P(X = 3) = C(5, 3) \cdot (0.6)^3 \cdot (0.4)^2 = 10 \cdot 0.216 \cdot 0.16 = 10 \cdot 0.03456 = 0.3456$. So P(exactly 3 pass) = 0.3456 (i.e. about 34.56%).
Step 3 [2 marks]	**Mean and variance**	Mean $E(X) = n\cdot p = 5 \times 0.6 = 3$. Variance $Var(X) = n\cdot p\cdot(1-p) = 5 \times 0.6 \times 0.4 = 1.2$. Standard deviation $\sigma = \sqrt{1.2} \approx 1.095$. Conclusion: on average 3 students out of 5 pass, with variance 1.2.

COMMON LOSS OF MARKS:

- Forgetting the C(5, 3) = 10 factor and giving P(X = 3) = 0.216 × 0.16 = 0.03456 — that's only one of the ten arrangements.
- Computing 0.6^3 as 1.8 instead of 0.216 — three decimal multiplications are the most error-prone step.
- Using mean = n + p = 5.6 or variance = n × p = 3; the correct formulas are np and np(1−p).

★ TOPPER TEMPLATE — 4 marks: 'The number of defective bulbs in a batch follows a Poisson distribution with mean λ = 2. Find P(X = 0) and P(X ⩾ 1). Use e^(−2) = 0.1353.'

Most years

Step 1 [1 mark]	**State the distribution**	X ~ Poisson(λ = 2). PMF: $P(X = k) = e^{-\lambda} \cdot \lambda^k / k! = e^{-2} \cdot 2^k / k!$. Given $e^{-2} = 0.1353$.
Step 2 [1 mark]	**P(X = 0)**	$P(X = 0) = e^{-2} \cdot 2^0 / 0! = 0.1353 \cdot 1 / 1 = 0.1353$. So the probability of NO defective bulbs is 0.1353 (≈ 13.53%).
Step 3 [1 mark]	**P(X ⩾ 1) by complement**	$P(X \geqslant 1) = 1 - P(X = 0) = 1 - 0.1353 = 0.8647$.
Step 4 [1 mark]	**Interpretation**	So 13.53% of batches will have zero defective bulbs, and 86.47% will have at least one defective bulb. This kind of rare-count modelling is the natural use of Poisson in quality control.

COMMON LOSS OF MARKS:

- Computing P(X ⩾ 1) as P(X = 1) only — that misses X = 2, 3, ⋯ Always use the complement 1 − P(X = 0).
- Forgetting that 0! = 1 — students sometimes treat it as 0 and get a divide-by-zero panic.
- Misreading the e^(−2) value or trying to compute it without the given hint.

★ TOPPER TEMPLATE — 4 marks: 'Heights of adult males are normally distributed with μ = 170 cm and σ = 8 cm. Find P(X ⩽ 178). Use $\Phi(1.00) = 0.8413$.'

Most years (2022 onwards)

Step 1 **[1 mark]**	**State the distribution and standardise**	$X \sim N(\mu = 170, \sigma = 8)$. To use the standard normal table, standardise: $z = (X - \mu)/\sigma$. For X = 178: z = (178 − 170)/8 = 8/8 = 1.00.
Step 2 **[1 mark]**	**Convert to standard normal probability**	$P(X \leqslant 178) = P(Z \leqslant 1.00) = \Phi(1.00)$, where Z is the standard normal variable.
Step 3 **[1 mark]**	**Read from the z-table**	From the given table value, $\Phi(1.00) = 0.8413$.
Step 4 **[1 mark]**	**State the answer with interpretation**	Therefore $P(X \leqslant 178) = 0.8413$, i.e. approximately 84.13% of adult males are 178 cm or shorter. (Consistent with the 68% rule: $178 = \mu + \sigma$, so about (100% − 68%)/2 = 16% lie above and 84% lie below — matching 0.8413.)

COMMON LOSS OF MARKS:

- Forgetting to standardise — students plug 178 directly into the z-table.
- Using $P(Z > z) = \Phi(z)$ — $\Phi(z)$ is the LEFT tail; the right tail is $1 - \Phi(z)$.
- Dividing by σ^2 instead of σ in the z-formula.

MARKING SCHEME — GENERAL NOTES

- On distribution-table questions, ALWAYS verify $\Sigma p_i = 1$ — a mark is reserved for this check.
- On variance questions, use $Var(X) = E(X^2) - [E(X)]^2$; tabulate x, p, xp, x^2p as separate columns.
- On binomial questions, NEVER omit the C(n, k) factor; mean = np and variance = np(1−p).
- On Poisson questions, use given $e^{(-\lambda)}$ values; remember $P(X \geqslant 1) = 1 - P(X = 0)$; mean = variance = λ.
- On normal-distribution questions, STANDARDISE $z = (X - \mu)/\sigma$ first; read $\Phi(z)$ for left-tail probabilities; use $1 - \Phi(z)$ for right-tail; symmetric values via $\Phi(-z) = 1 - \Phi(z)$.