CHAPTER 4

Probability Distributions

CBSE Class 12 Applied Mathematics · Unit 4

CBSE · Applied Mathematics · Class 12

WHAT THIS CHAPTER DOES

A Construct a probability distribution table from a small random experiment and verify $\Sigma p_i = 1$.

B Compute the expected value E(X) and variance Var(X) of any discrete distribution.

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's mission

1. Construct a probability distribution table from a small random experiment and verify $\Sigma p_i = 1$.
2. Compute the expected value E(X) and variance Var(X) of any discrete distribution.
3. Apply the binomial PMF $P(X = k) = C(n,k)\, p^k (1-p)^{(n-k)}$ and its mean np and variance np(1−p).
4. Use the Poisson PMF $e^{(-\lambda)} \lambda^k / k!$ and the normal-distribution z-score $z = (X - \mu)/\sigma$ with the standard table.

WHY THIS MATTERS

Why this chapter matters

1. Worth ~8 marks — predictable easy marks if formulas are memorised, predictable lost marks if not.
2. Probability distributions are the LANGUAGE of risk and quality control in business, finance and engineering.
3. Every later statistics topic — sampling, confidence intervals, hypothesis testing — is built on the binomial and normal distributions introduced here.

TOPIC

A

Random variables & probability distributions

THEOREM · LOAD-BEARING RESULT

Random variable and probability distribution

> A random variable X assigns a numerical value to each outcome of a random experiment. A DISCRETE random variable takes countably many values; its PMF $p_i = P(X = x_i)$ satisfies $0 \leqslant p_i \leqslant 1$ and $\Sigma p_i = 1$. The list of (x_i, p_i) pairs is the probability distribution of X.

STATEMENT

Discrete X has PMF $p_i = P(X = x_i)$ with: (i) $0 \leqslant p_i \leqslant 1$, (ii) $\Sigma p_i = 1$. Worked: X = number of heads in two fair coin tosses, $X \in \{0,1,2\}$ with P = 1/4, 1/2, 1/4 — verify 1/4 + 1/2 + 1/4 = 1 ✓.

WHY THIS MATTERS

- The distribution table is the foundation of the unit
- Every expected-value, variance and named-distribution question begins from such a table.

WATCH OUT FOR

NOTE Every probability in [0,1] AND total exactly 1. When a table has an unknown p, solve for it using $\Sigma = 1$. Forgetting to verify $\Sigma p_i = 1$ is the #1 source of mark losses.

WORKED EXAMPLE

Worked example — distribution from two coin tosses

1. QUESTION: Two fair coins are tossed. Let X = number of heads. Construct the probability distribution of X.
2. STEP 1 — Sample space: $S = \{HH, HT, TH, TT\}$, each outcome with probability $1/4$ (independent fair coins).
3. STEP 2 — Values of X: $X = 0$ only on TT; $X = 1$ on HT or TH (two outcomes); $X = 2$ only on HH.
4. STEP 3 — Compute probabilities: $P(X = 0) = 1/4$; $P(X = 1) = 2/4 = 1/2$; $P(X = 2) = 1/4$.
5. STEP 4 — Verify: $1/4 + 1/2 + 1/4 = 1$ ✓. The probability distribution is $X = 0, 1, 2$ with $P = 1/4, 1/2, 1/4$. This is the table every subsequent calculation will use.

TOPIC

B

Expected value & variance

THEOREM · LOAD-BEARING RESULT

Expected value, variance and standard deviation

> Expected value $E(X) = \Sigma x_i \cdot p_i$ is the long-run average. Variance $Var(X) = E(X^2) - [E(X)]^2$, where $E(X^2) = \Sigma x_i^2 \cdot p_i$. SD $\sigma = \sqrt{Var(X)}$ measures spread in the units of X.

STATEMENT

$\mu = E(X) = \Sigma x_i p_i$. $E(X^2) = \Sigma x_i^2 p_i$. $Var = E(X^2) - \mu^2$. Equivalent: $Var = \Sigma (x_i - \mu)^2 p_i$. SD $\sigma = \sqrt{Var}$. Worked: X with P(0,1,2) = 1/4,1/2,1/4: $E(X) = 1$; $E(X^2) = 0 + 1/2 + 1 = 1.5$; $Var = 0.5$; $\sigma \approx 0.707$.

WHY THIS MATTERS

- Mean and variance compress a whole distribution into two numbers
- Almost every board question asks for one or both
- In finance: mean = expected return, σ = risk.

WATCH OUT FOR

NOTE Variance is $E(X^2) - \mu^2$, NOT $E(X) - \mu^2$ and NOT $\Sigma (x_i - \mu) p_i$ (which is 0 by definition). The square is essential.

WORKED EXAMPLE

Worked example — E(X) and Var(X) from a table

1. QUESTION: X has distribution $P(X = 1) = 0.2$, $P(X = 2) = 0.5$, $P(X = 3) = 0.3$. Find E(X), Var(X) and σ.
2. STEP 1 — Check Σp_i: $0.2 + 0.5 + 0.3 = 1.0$ ✓.
3. STEP 2 — $E(X) = \Sigma x_i \cdot p_i = 1(0.2) + 2(0.5) + 3(0.3) = 0.2 + 1.0 + 0.9 = 2.1$.
4. STEP 3 — $E(X^2) = \Sigma x_i^2 \cdot p_i = 1(0.2) + 4(0.5) + 9(0.3) = 0.2 + 2.0 + 2.7 = 4.9$.
5. STEP 4 — $Var(X) = E(X^2) - [E(X)]^2 = 4.9 - (2.1)^2 = 4.9 - 4.41 = 0.49$. $\sigma = \sqrt{0.49} = 0.7$.
6. ANSWER: $E(X) = 2.1$, $Var(X) = 0.49$, $\sigma = 0.7$. Tabulate columns x, p, xp, x^2p to keep arithmetic clean — this is the marking-scheme expected layout.

TOPIC

C

Bernoulli trials & the binomial distribution

THEOREM · LOAD-BEARING RESULT

The binomial distribution B(n, p)

> A Bernoulli trial has two outcomes — success (probability p) and failure (1−p). If X counts the number of successes in n INDEPENDENT Bernoulli trials, X ~ B(n, p) with PMF P(X = k) = C(n, k) p^k (1−p)^(n−k). Mean np; variance np(1−p).

STATEMENT

X ~ B(n, p): P(X = k) = C(n, k) p^k (1−p)^(n−k) for k = 0, ⋯, n. E(X) = np. Var(X) = np(1−p). SD √[np(1−p)]. Conditions: fixed n, independent trials, constant p. Worked: B(5, 0.4), P(X = 2) = C(5,2)·0.4^2·0.6^3 =

WHY THIS MATTERS

- The binomial is the most-tested distribution in Class 12 Applied Maths (~28% of unit marks)
- Uses: defects per batch, customer conversions, pass-counts — any 'k out of n' problem with replacement.

WATCH OUT FOR

NOTE Binomial needs INDEPENDENT trials with CONSTANT p — 'without replacement' breaks both, so it's NOT binomial (hypergeometric). Never omit the C(n, k) factor — p^k(1−p)^(n−k) alone is ONE ordering, not k successes in any order.

WORKED EXAMPLE

Worked example — binomial B(5, 0.4)

1. QUESTION: A salesperson closes a deal with probability 0.4 per call, independently. They make 5 calls. Find P(exactly 2 sales) and the mean and variance.
2. STEP 1 — Identify: $X \sim B(n = 5, p = 0.4)$, so $q = 1 - p = 0.6$.
3. STEP 2 — PMF: $P(X = 2) = C(5, 2) \cdot (0.4)^2 \cdot (0.6)^3$.
4. STEP 3 — Evaluate factors: $C(5, 2) = 10$; $(0.4)^2 = 0.16$; $(0.6)^3 = 0.216$.
5. STEP 4 — Multiply: $P(X = 2) = 10 \times 0.16 \times 0.216 = 10 \times 0.03456 = 0.3456$.
6. STEP 5 — Mean and variance: $E(X) = np = 5 \times 0.4 = 2.0$. $Var(X) = np(1 - p) = 5 \times 0.4 \times 0.6 = 1.2$. $\sigma = \sqrt{1.2} \approx 1.095$.
7.

TOPIC

D

Poisson distribution

THEOREM · LOAD-BEARING RESULT

The Poisson distribution Po(λ)

> The Poisson distribution Po(λ) models the count of RARE events occurring in a fixed interval of time or space at average rate $\lambda > 0$. PMF: $P(X = k) = e^{-\lambda} \cdot \lambda^k / k!$ for $k = 0, 1, 2, \cdots$. Both the mean and the variance equal λ.

STATEMENT

$X \sim Po(\lambda)$: $P(X = k) = e^{-\lambda} \cdot \lambda^k / k!$ for $k = 0, 1, 2, \cdots$. $E(X) = \lambda$. $Var(X) = \lambda$. SD $\sigma = \sqrt{\lambda}$. Poisson is the LIMITING distribution of B(n, p) as $n \to \infty$, $p \to 0$ with $np = \lambda$ fixed. Worked: $Po(\lambda = 3)$, $P(X = 2) =$

WHY THIS MATTERS

- Poisson reliably contributes a 4-mark question and is the workhorse model in quality control (defects per batch), telecom (calls per minute), and operations (customer arrivals)
- Recognising the right scenarios — rare events, fixed interval, given average rate — is half the battle.

WATCH OUT FOR

NOTE Use Poisson when there is NO fixed n but only an average rate λ over an interval. Use binomial when there IS a fixed n and a per-trial probability p. The two are NOT interchangeable, though Poisson approximates Binomial for large n and small p with $\lambda = np$.

WORKED EXAMPLE

Worked example — Poisson Po(3)

1. QUESTION: The number of customer arrivals at a counter per minute follows Po(λ = 3). Find P(X = 2) and P(X ⩾ 1). Use e^(−3) = 0.0498.
2. STEP 1 — Identify: X ~ Po(λ = 3). PMF: P(X = k) = e^(−3) · 3^k / k!.
3. STEP 2 — P(X = 2) = e^(−3) · 3^2 / 2! = 0.0498 · 9 / 2 = 0.0498 · 4.5 = 0.2240.
4. STEP 3 — P(X = 0) = e^(−3) · 3^0 / 0! = 0.0498 · 1 / 1 = 0.0498.
5. STEP 4 — P(X ⩾ 1) by complement: 1 − P(X = 0) = 1 − 0.0498 = 0.9502.
6. ANSWER: P(X = 2) ≈ 0.2240 (≈ 22.4%); P(X ⩾ 1) ≈ 0.9502 (≈ 95.02%). Mean and variance both equal λ = 3.

TOPIC

E

Normal distribution

THEOREM · LOAD-BEARING RESULT

The normal distribution $N(\mu, \sigma^2)$

> A continuous random variable X is normally distributed $N(\mu, \sigma^2)$ if its density is a bell-shaped curve symmetric about μ with spread σ. About 68% of values lie within 1σ of μ, 95% within 2σ, and 99.7% within 3σ. Probabilities are computed by standardising $z = (X - \mu)/\sigma$ and reading $\Phi(z) = P(Z \leqslant z)$ from the standard normal table.

STATEMENT

$X \sim N(\mu, \sigma^2)$; $Z = (X - \mu)/\sigma \sim N(0, 1)$ is the standard normal. $\Phi(z) = P(Z \leqslant z)$ is tabulated. Useful values: $\Phi(1.00) = 0.8413$; $\Phi(1.96) = 0.9750$; $\Phi(2.00) = 0.9772$; $\Phi(2.58) = 0.9950$. By

WHY THIS MATTERS

- Normal-distribution z-look-ups have become a reliable 4-mark question since 2022
- Beyond marks, the normal distribution is THE central distribution of statistics — almost every measurement, exam score, and average behaves approximately normally for large samples (Central

WATCH OUT FOR

NOTE $z = (X - \mu)/\sigma$ is the NUMBER OF STANDARD DEVIATIONS — it is NOT a probability. Convert it to a probability by reading $\Phi(z)$ from the table. Also: $\Phi(z)$ is the LEFT tail; for the right tail use $1 - \Phi(z)$; between a and b use $\Phi(b) - \Phi(a)$. Memorise $\Phi(1) \approx 0.84$, $\Phi(2) \approx 0.975$, $\Phi(1.96) = 0.975$ as sanity anchors.

WORKED EXAMPLE

Worked example — N(170, 8^2) z-table look-up

1. QUESTION: Heights of adult males are N(μ = 170 cm, σ = 8 cm). Find $P(X \leqslant 178)$ and $P(X > 186)$. Use $\Phi(1.00) = 0.8413$, $\Phi(2.00) = 0.9772$.
2. STEP 1 — Standardise X = 178: $z = (178 - 170)/8 = 8/8 = 1.00$. So $P(X \leqslant 178) = P(Z \leqslant 1.00) = \Phi(1.00) = 0.8413$.
3. STEP 2 — Standardise X = 186: $z = (186 - 170)/8 = 16/8 = 2.00$. So $P(X \leqslant 186) = \Phi(2.00) = 0.9772$.
4. STEP 3 — Right-tail by complement: $P(X > 186) = 1 - \Phi(2.00) = 1 - 0.9772 = 0.0228$.
5. ANSWER: $P(X \leqslant 178) = 0.8413$ (about 84%); $P(X > 186) = 0.0228$ (about 2.28%). Consistent with the empirical rule: $178 = \mu + \sigma \Rightarrow$ ~84% below; $186 = \mu + 2\sigma \Rightarrow$ ~97.7% below, ~2.3% above.

TOPIC

Choosing the right distribution

BINOMIAL WHEN N IS FIXED AND TRIALS ARE INDEPENDENT

When the question gives a FIXED number n of independent trials each with the SAME success probability p ('out of 10 customers', 'in 8 throws', 'a salesperson makes 5 calls') and counts how many succeed, use B(n, p). PMF: C(n, k) p^k (1−p)^(n−k); mean np; variance np(1−p). Dead

POISSON WHEN ONLY A RATE Λ IS GIVEN

When the problem mentions an average RATE per unit time or space ('on average 3 customers per minute', 'mean 2 defects per batch') and asks for the probability of exactly k events in one such interval, use Po(λ). PMF e^(−λ) λ^k / k!; mean = variance = λ. Trigger

NORMAL FOR CONTINUOUS MEASUREMENTS

When the variable is CONTINUOUS (heights, weights, exam scores, lifetimes) with given mean μ and SD σ, use N(μ, σ^2). STANDARDISE z = (X − μ)/σ, then look up Φ(z). Use Φ(z) for left-tail (P(X ⩽ a)), 1 − Φ(z) for right-tail (P(X > a)), Φ(b) − Φ(a) for between. Sanity-check

READ CAREFULLY — THE WORDING PICKS THE MODEL

The commonest modelling mistake is forcing binomial onto a Poisson scenario (or vice versa). 'How many out of 12' → binomial; 'how many per hour' → Poisson; 'what proportion are taller than' → normal; if WITHOUT replacement, binomial does NOT apply. Five seconds spent

TOPIC

Probability distribution — sum of probabilities

TRAP → TRUTH

× **MISTAKE** The probabilities in a distribution can take any positive values.

✓ **CORRECT** For ANY valid discrete probability distribution, every p_i must satisfy $0 \leqslant p_i \leqslant 1$ AND $\Sigma\ p_i = 1$. If a table is given with an unknown probability, you almost always solve for it using $\Sigma\ p_i = 1$. Forgetting this constraint is the #1 reason students give 'distributions' that aren't valid distributions.

TOPIC

Variance formula

TRAP → TRUTH

× **MISTAKE** $Var(X) = E(X) - [E(X)]^2$, or $Var(X) = \Sigma (x_i - \mu) \cdot p_i$.

✓ **CORRECT** Variance $Var(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \Sigma x_i^2 \cdot p_i$. Equivalently, $Var(X) = \Sigma (x_i - \mu)^2 \cdot p_i$ (note the SQUARE on the deviation, and the SUM weighted by p_i). The deviations $(x_i - \mu)$ without squaring sum to zero by definition — they always do — so that 'formula' gives 0 for every distribution and cannot be right.

TOPIC

Binomial vs Poisson

TRAP → TRUTH

× **MISTAKE** Binomial and Poisson are interchangeable for counting problems.

✓ **CORRECT** BINOMIAL B(n, p) applies when there is a FIXED number of trials n and each trial independently succeeds with probability p (e.g. 'in 10 tosses of a fair coin, P(exactly 4 heads)'). POISSON applies to counts of RARE events in a fixed interval of time or space at average rate λ, with no fixed n (e.g. 'the number of customer arrivals per hour, mean 5'). Use Poisson as an approximation to Binomial only when n is large and p is small with $\lambda = np$ finite.

TOPIC

With vs without replacement

TRAP → TRUTH

× **MISTAKE** Drawing balls 'without replacement' is still a binomial experiment.

✓ **CORRECT** Binomial requires INDEPENDENT trials with a CONSTANT success probability p. Without replacement, the probability of success changes after each draw (the population shrinks), so the trials are NOT independent and the count is NOT binomial — it follows a hypergeometric distribution. Replace-and-redraw experiments ARE binomial; sample-and-keep experiments are not.

TOPIC

Mean and variance of B(n, p)

TRAP → TRUTH

× **MISTAKE** For B(n, p), mean = n + p and variance = n − p.

✓ **CORRECT** For X ~ B(n, p): mean E(X) = n·p, variance Var(X) = n·p·(1−p), and standard deviation $\sigma = \sqrt{[n \cdot p \cdot (1-p)]}$. Example: for B(10, 0.5), mean = 10×0.5 = 5 and variance = 10×0.5×0.5 = 2.5. Memorise the products np and np(1−p) — they are the most-tested formulas of the unit.

TOPIC

Standard normal — what z measures

TRAP → TRUTH

× **MISTAKE** The z-score equals the probability.

✓ **CORRECT** The z-score $z = (X - \mu)/\sigma$ is the number of STANDARD DEVIATIONS the value X lies from the mean — it is NOT a probability. To turn a z-score into a probability you must look it up in the standard normal table, which gives $\Phi(z) = P(Z \leqslant z)$, the area to the LEFT of z under the standard normal curve. $P(Z > z) = 1 - \Phi(z)$; $P(a \leqslant Z \leqslant b) = \Phi(b) - \Phi(a)$.

TOPIC

Empirical 68-95-99.7 rule

TRAP → TRUTH

× **MISTAKE** About 68% of any data lies within 1 standard deviation of the mean.

✓ **CORRECT** The 68-95-99.7 rule applies ONLY to the NORMAL distribution. For a normal random variable, approximately 68% of values lie within 1σ of the mean μ, 95% within 2σ, and 99.7% within 3σ. For other distributions (skewed, bimodal, uniform) these percentages are wrong. Always confirm that the variable is (approximately) normal before quoting the rule.

TOPPER TEMPLATE · MARK-BY-MARK

3 marks: 'Two fair coins are tossed. Let X be the number of heads. Construct the probability

1	**LIST SAMPLE SPACE AND VALUES OF X** 1 m	The sample space of two fair coin tosses is S = {HH, HT, TH, TT}, each with probability 1/4. X = number of heads can take values 0, 1, 2. X = 0 occurs only on TT (prob 1/4); X = 1 occurs on HT or TH (prob 2/4 = 1/2); X = 2 occurs only on HH (prob 1/4).
2	**WRITE THE PROBABILITY DISTRIBUTION TABLE** 1 m	Distribution table: X = 0, 1, 2 with P(X) = 1/4, 1/2, 1/4. Check: 1/4 + 1/2 + 1/4 = 1 ✓ (sum of probabilities = 1).
3	**COMPUTE E(X)** 1 m	$E(X) = \Sigma x_i \cdot p_i = 0 \cdot (1/4) + 1 \cdot (1/2) + 2 \cdot (1/4) = 0 + 0.5 + 0.5 = 1$. So the expected number of heads in two tosses is 1.

TOPPER TEMPLATE · MARK-BY-MARK

5 marks: 'The probability that a student passes a test is 0.6. Five students take the test mean and variance of the number who pass.

1 IDENTIFY THE DISTRIBUTION (1 m)

Let X = number of students who pass out of n = 5. Each trial is independent with success probability p = 0.6 (and failure probability q = 1 − p = 0.4). So X ~ B(5, 0.6) — a binomial distribution. PMF: P(X = k) = C(5, k) (0.6)^k (0.4)^(5−k).

2 COMPUTE P(X = 3) (2 m)

P(X = 3) = C(5, 3) · $(0.6)^3$ · $(0.4)^2$ = 10 · 0.216 · 0.16 = 10 · 0.03456 = 0.3456. So P(exactly 3 pass) = 0.3456 (i.e. about 34.56%).

3 MEAN AND VARIANCE (2 m)

Mean E(X) = n·p = 5 × 0.6 = 3. Variance Var(X) = n·p·(1−p) = 5 × 0.6 × 0.4 = 1.2. Standard deviation σ = √1.2 ≈ 1.095. Conclusion: on average 3 students out of 5 pass, with variance 1.2.

TOPPER TEMPLATE · MARK-BY-MARK

4 marks: 'The number of defective bulbs in a batch follows a Poisson distribution with mean λ

1 STATE THE DISTRIBUTION (1 m)

X ~ Poisson(λ = 2). PMF: P(X = k) = e^(−λ) · λ^k / k! = e^(−2) · 2^k / k!. Given e^(−2) = 0.1353.

2 P(X = 0) (1 m)

P(X = 0) = e^(−2) · 2^0 / 0! = 0.1353 · 1 / 1 = 0.1353. So the probability of NO defective bulbs is 0.1353 (≈ 13.53%).

3 P(X ⩾ 1) BY COMPLEMENT (1 m)

P(X ⩾ 1) = 1 − P(X = 0) = 1 − 0.1353 = 0.8647.

4 INTERPRETATION (1 m)

So 13.53% of batches will have zero defective bulbs, and 86.47% will have at least one defective bulb. This kind of rare-count modelling is the natural use of Poisson in quality control.

PYQ PATTERNS

Top PYQ patterns to drill

#1	Construct a probability distribution table from a small experiment (e.g. tossing two coins, drawing a card) and verify Σ p_i = 1. (2-3 marks)	Annual
#2	Compute E(X) and Var(X) from a given probability distribution table. (3 marks)	Annual
#3	Find P(X = k), or P(X ⩾ k), for a binomial random variable B(n, p)	most often with small n and round p. (5 marks) — Annual
#4	Apply the Poisson distribution to a rare-event count (e.g. defective items, accidents) with a given λ. (4-5 marks)	2022, 2023, 2024, 2025
#5	Standardise a normal variable using $z = (X - \mu)/\sigma$ and read the probability from the standard normal table. (4 marks)	2022, 2024, 2025

MARKS DISTRIBUTION

10-year marks distribution

10-YEAR PYQ MARKS DISTRIBUTION

Random variable & probability distribution table	10%
Expected value and variance	14%
Binomial distribution B(n,p)	22%
Poisson distribution	12%
Normal distribution & z-table	14%
Definition / formula MCQs	8%

RECAP · MEMORISE THESE

Recap — the probability-distribution spine

1 Distribution table — $0 \leqslant p_i \leqslant 1$ and $\Sigma p_i = 1$. Always verify the sum before any computation.

2 Expectation & variance — $E(X) = \Sigma x_i p_i$; $Var(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \Sigma x_i^2 p_i$; $\sigma = \sqrt{Var}$.

3 Binomial $B(n, p)$ — $P(X = k) = C(n, k) p^k (1-p)^{(n-k)}$; mean np; variance $np(1-p)$. Fixed n, independent trials.

4 Poisson $Po(\lambda)$ — $P(X = k) = e^{(-\lambda)} \lambda^k / k!$; mean = variance = λ. Rare events in a fixed interval.

5 Normal $N(\mu, \sigma^2)$ — Standardise $z = (X - \mu)/\sigma$; read $\Phi(z)$ from the standard normal

WHAT'S NEXT

What's next

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- → Chapter 5 — Inferential Statistics (sampling distributions, confidence intervals, t-tests).
- → Sit the 15-MCQ Quick Drill for this Probability Distributions chapter.
- → Then the full Board-Pattern Paper — 28 marks.

You've mastered Unit 4 — the mathematics of uncertainty.

Random variables · binomial · Poisson · normal — now prove it on the board paper.

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