Ready For Boards
10th & 12th Exam Prep

CHAPTER 1

Relations and Functions

CBSE Class 12 · Mathematics · Chapter 1

CBSE · Mathematics · Class 12

WHAT THIS CHAPTER DOES

A Classify any relation as reflexive, symmetric, transitive, or equivalence — with proof.

B Prove a function is one-one, onto, or bijective — and find its inverse.

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's mission

1. Classify any relation as reflexive, symmetric, transitive, or equivalence — with proof.
2. Prove a function is one-one, onto, or bijective — and find its inverse.
3. Compose functions correctly: $(fog)(x) = f(g(x))$.
4. Score 5/6 marks on this chapter's slice of the board paper.

WHY THIS MATTERS

Why this chapter matters

1. 4-6 marks every CBSE board paper.
2. Foundation for every later chapter (Continuity, Differentiability, Integrals).
3. Equivalence-relation proofs and bijection problems are predictable big-mark gifts.

TOPIC

A

Relations — definitions & types

THEOREM · LOAD-BEARING RESULT

Equivalence Relation

> *A relation R on set A is an equivalence relation if and only if it is reflexive, symmetric, AND transitive.*

STATEMENT

Reflexive: $\forall a \in A, (a,a) \in R$. Symmetric: $(a,b) \in R \Rightarrow (b,a) \in R$. Transitive: $(a,b) \in R \wedge (b,c) \in R \Rightarrow (a,c) \in R$. Equivalence relations partition A into disjoint equivalence classes: $[a] = \{x \in A : (a,x) \in R\}$.

WHY THIS MATTERS

- Equivalence relations capture the intuition of 'same kind' — they partition a set into mutually exclusive, exhaustive groups
- Examples: 'has the same parity as' on integers (2 classes — even, odd)
- 'is congruent to mod n' (n classes)

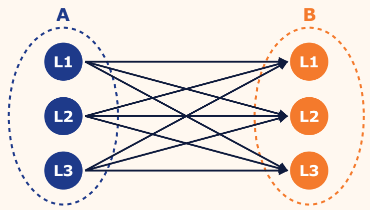
WATCH OUT FOR

NOTE DON'T skip reflexive — most common mark loss. ALL THREE conditions are required; missing one breaks equivalence.

WORKED EXAMPLE

Prove: 'is parallel to' on lines is equivalence

1. Reflexive: every line is parallel to itself ⇒ $(L, L) \in R \ \forall L$. ✓
2. Symmetric: if $L_1 \parallel L_2$, then $L_2 \parallel L_1$ (parallelism is symmetric by definition). ✓
3. Transitive: if $L_1 \parallel L_2$ and $L_2 \parallel L_3$, then $L_1 \parallel L_3$ (transitivity of parallel lines). ✓
4. All three hold ⇒ R is an equivalence relation. Equivalence classes = sets of mutually parallel lines (each direction is its own class).



R = 'is parallel to' (full diagonal + symmetric closure)

TOPIC

B

Functions — one-one, onto, bijective

THEOREM · LOAD-BEARING RESULT

Bijective Function

A function f: A → B is bijective iff it is both one-one (injective) and onto (surjective).

STATEMENT

One-one: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ (different inputs → different outputs). Onto: Range(f) = Codomain. Equivalently: $\forall\, y \in B, \exists\, x \in A$ with $f(x) = y$. Bijective = one-one AND onto. ONLY bijective functions have inverses.

WHY THIS MATTERS

- Bijection captures the idea of a 'perfect pairing': every element of A pairs with exactly one of B, with no leftovers on either side
- Inverse functions exist precisely because of this perfect pairing.

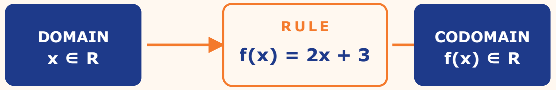
WATCH OUT FOR

NOTE Drawing the arrow diagram helps: one-one means no two arrows hit the same target; onto means every target gets hit at least once.

WORKED EXAMPLE

Show f(x) = 2x + 3 is bijective on R

1. One-one: assume $f(x_1) = f(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$. ✓ So f is one-one.
2. Onto: take $y \in R$. Solve $f(x) = y \Rightarrow 2x + 3 = y \Rightarrow x = (y - 3)/2$. This $x \in R$ always ✓. So f is onto.
3. Both ⇒ f is bijective. Inverse: $f^{-1}(x) = (x - 3)/2$.



TOPIC

C

Composition & Inverse

TOPIC

Composing two functions

DEFINITION

Function composition (fog)(x) = f(g(x)) means: apply g first to x, then apply f to the result. The domain of fog is the set of x in domain(g) for which g(x) lies in domain(f) — both functions must accept their respective inputs. For composition to be defined at all, range(g) ⊆ domain(f). Example: $f(x)=\sqrt{x}$ with domain $x \geq 0$, and $g(x)=x-4$ with domain ℝ

NOT COMMUTATIVE

Composition is NOT commutative in general — fog ≠ gof. Classic counterexample: take $f(x) = x^2$ and $g(x) = x + 1$. Then $(fog)(x) = f(g(x)) = (x+1)^2 = x^2 + 2x + 1$, but $(gof)(x) = g(f(x)) = x^2 + 1$. The two expressions differ for every $x \neq 0$. Composition is commutative only in special cases — for instance when one of the functions is the

ASSOCIATIVE

Composition IS associative: (fog)oh = fo(goh) for any three functions f, g, h whose compositions are defined. This means that to compute f(g(h(x))) you may bracket either way: first compose goh and then apply f, or first compose fog and then apply it to h(x). The result is identical. Associativity is what allows mathematicians to write

IDENTITY

The identity function I(x) = x maps every element to itself. For any function f: A → A, composition with the identity is a no-op: foI = Iof = f. This is the defining property of the identity element under composition. The identity function is also the only function that is its own inverse ($I = I^{-1}$), and it serves as the test for whether a candidate g is

THEOREM · LOAD-BEARING RESULT

Inverse Function

> *A function f: A → B has an inverse f^{-1}: B → A iff f is bijective.*

STATEMENT

f^{-1} is defined by: $f^{-1}(y) = x \Leftrightarrow f(x) = y$. Then fof^{-1} = I_B and $f^{-1}of$ = I_A. Crucially: $(gof)^{-1} = f^{-1}og^{-1}$ — the order REVERSES.

WHY THIS MATTERS

- Inverse functions 'undo' the original
- Decryption is the inverse of encryption
- Square root is the inverse of squaring (restricted to non-negatives).

WATCH OUT FOR

NOTE $(gof)^{-1} = f^{-1}og^{-1}$, NOT $f^{-1}og^{-1}$ in different order. Same as putting on socks then shoes — to reverse, take off shoes then socks.

TOPIC

Reflexive vs symmetric

TRAP → TRUTH

× **MISTAKE** Reflexive means (a,a) is in R for SOME a in A.

✓ **CORRECT** Reflexive means $(a,a) \in R$ for EVERY $a \in A$. Just one a satisfying it is NOT enough. The universal quantifier matters.

TOPIC

Equivalence relation

TRAP → TRUTH

× **MISTAKE** An equivalence relation needs only to be symmetric and transitive.

✓ **CORRECT** An equivalence relation needs ALL THREE: reflexive AND symmetric AND transitive. Drop any one and it's not equivalence. Forgetting reflexive is the most common −1 mark.

TOPIC

One-one (injective)

TRAP → TRUTH

× **MISTAKE** f is one-one if every element of codomain has a pre-image.

✓ **CORRECT** f is one-one (injective) if DIFFERENT inputs give DIFFERENT outputs: $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$. The 'every element of codomain has a pre-image' is the definition of ONTO, not one-one. Confusing the two is a 2-mark loss.

TOPIC

Onto (surjective)

TRAP → TRUTH

✗ **MISTAKE** f is onto if its range equals its domain.

✓ **CORRECT** f is onto if its RANGE equals its CODOMAIN (not its domain). Range = set of actual outputs; codomain = set of allowed outputs. Range $\subseteq$ codomain always; onto means Range = Codomain.

TOPIC

Inverse of composition

TRAP → TRUTH

× **MISTAKE** $(fog)^{-1} = f^{-1}og^{-1}$.

✓ **CORRECT** $(fog)^{-1} = g^{-1}of^{-1}$. The order REVERSES when you take the inverse of a composition. Order matters in composition (gof ≠ fog in general) — and matters DOUBLY when inverting.

TOPIC

Inverse function existence

TRAP → TRUTH

× **MISTAKE** Every function has an inverse.

✓ **CORRECT** A function has an inverse ONLY if it is BIJECTIVE (both one-one AND onto). $f(x) = x^2$ ($R \to R$) is NOT bijective (not one-one), so no inverse. But $f(x) = x^2$ restricted to $[0,\infty) \to [0,\infty)$ IS bijective ⇒ has inverse $\sqrt{x}$.

TOPIC

Identity vs inverse

TRAP → TRUTH

× **MISTAKE** If $foI = f$, then I is the inverse of f.

✓ **CORRECT** If $foI = f$ and $Iof = f$, then I is the IDENTITY function (not the inverse). The inverse is f^{-1} such that $fof^{-1} = f^{-1}of = I$ (identity). Identity ≠ inverse.

TOPPER TEMPLATE · MARK-BY-MARK

5-mark: 'Show that R on set A is an equivalence relation. List equivalence classes.'

Step	Criterion	Marks	What to write
1	**REFLEXIVE: ∀ A ∈ A, (A,A) ∈ R**	1 m	Take arbitrary a ∈ A. Show (a, a) satisfies the relation's definition. State 'so R is reflexive.'
2	**SYMMETRIC: (A,B) ∈ R ⇒ (B,A) ∈ R**	1 m	Take arbitrary (a,b) ∈ R. Use the relation definition. Show (b,a) also satisfies it. State 'so R is symmetric.'
3	**TRANSITIVE: (A,B) ∈ R AND (B,C) ∈ R ⇒ (A,C) ∈ R**	2 m	Take arbitrary (a,b), (b,c) ∈ R. Use both relation definitions algebraically. Derive that (a,c) also satisfies. State 'so R is transitive.' 2 marks here because this step is the longest in most problems.
4	**CONCLUSION + LIST EQUIVALENCE CLASSES**	1 m	'Since R is reflexive, symmetric, and transitive, R is an equivalence relation.' List the classes: [a] = {x ∈ A : (a,x) ∈ R}, and show 2-3 representative classes.

TOPPER TEMPLATE · MARK-BY-MARK

5-mark: 'Show f: A → B is bijective. Find f^{-1}.'

1	**PROVE ONE-ONE** 2 m	Take $f(x_1) = f(x_2)$. Algebraically manipulate to derive $x_1 = x_2$. State 'so f is one-one.' 2 marks because this is core.
2	**PROVE ONTO** 1 m	Take arbitrary $y \in B$. Solve $f(x) = y$ for $x \in A$. Show $x \in A$ (it lies in domain). State 'so f is onto.'
3	**CONCLUDE BIJECTION** 1 m	'f is both one-one and onto ⇒ f is bijective ⇒ f^{-1} exists.'
4	**FIND F^{-1}** 1 m	From step 2's solving, you have x = (some expression in y). Replace y by x: $f^{-1}(x)$ = (that expression). State domain and codomain of f^{-1} explicitly.

TOPPER TEMPLATE · MARK-BY-MARK

3-mark: 'Find fog and gof, given f and g.'

1	**SET UP FOG** 1 m	(fog)(x) = f(g(x)). Substitute g(x) into f.
2	**SET UP GOF** 1 m	(gof)(x) = g(f(x)). Substitute f(x) into g. Simplify.
3	**COMPARE + STATE DOMAIN** 1 m	State whether fog = gof or not (usually NOT, in general). State the domain on which each is defined.

PYQ PATTERNS

Top PYQ patterns to drill

#1	Show that the relation R on set A defined by R = {(a,b): condition} is an equivalence relation. List equivalence classes. (5 marks)	Annual since 2017
#2	Show that the function f: A → B defined by f(x) = ... is (a) one-one (b) onto. Find f^{-1}. (5 marks)	2018, 2019, 2022, 2024
#3	If f and g are given, find fog and gof. Are they equal? (3 marks)	2018, 2020, 2023
#4	Check whether the given relation is reflexive / symmetric / transitive. (3 marks)	Most years 2017-2024
#5	Find the inverse of f(x) = (ax+b)/(cx+d). (3 marks)	2019, 2021, 2023

MARKS DISTRIBUTION

10-year marks distribution

10-YEAR PYQ MARKS DISTRIBUTION

Show a relation is equivalence (reflexive + symmetric + transitive)	38%
Show a function is one-one OR onto OR bijective	32%
Find inverse of a given bijective function	22%
Composition of functions (find gof, fog)	18%
Conceptual MCQ on type of relation/function	12%

RECAP · MEMORISE THESE

Recap

1 Relations — Reflexive ($\forall a, (a,a)$) + Symmetric ($(a,b) \Rightarrow (b,a)$) + Transitive ($(a,b) \wedge (b,c) \Rightarrow (a,c)$) = Equivalence.

2 Functions — One-one ($f(x_1)=f(x_2) \Rightarrow x_1=x_2$) + Onto (Range = Codomain) = Bijective. Only bijective ⇒ has inverse.

3 Composition — $(fog)(x) = f(g(x))$. $fog \neq gof$ in general. $(gof)^{-1} = f^{-1}og^{-1}$ (reverse order).

WHAT'S NEXT

What's next

- → Chapter 2 — Inverse Trigonometric Functions (builds on inverse-function ideas here).
- → Sit the 15-MCQ Quick Drill under 20-min timer. Target ≥ 12/15.
- → Then the full Board-Pattern Paper — 25 marks, 60 min.

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You've mastered the language of relations and functions.

Equivalence, bijection, inverse — now prove it.

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