ANSWER KEY & MARKING SCHEME · CBSE CLASS 12

Inverse Trigonometric Functions

Mathematics · Chapter 2 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — MCQ (1 mark each)

Q1. The principal value of $\sin^{-1}(-1/2)$ is: (a) $\pi/6$ (b) $-\pi/6$ (c) $5\pi/6$ (d) $-5\pi/6$ [1 mark]

Ans: (b) $-\pi/6$. $\sin(-\pi/6) = -1/2$ and $-\pi/6 \in [-\pi/2, \pi/2]$.

Q2. Range of $\sec^{-1} x$ is: (a) $[0, \pi]$ (b) $[0, \pi] - \{\pi/2\}$ (c) $(-\pi/2, \pi/2)$ (d) $[-\pi/2, \pi/2] - \{0\}$ [1 mark]

Ans: (b) $[0, \pi] - \{\pi/2\}$. $\pi/2$ is excluded because $\sec(\pi/2)$ is undefined.

Q3. $\sin^{-1} x + \cos^{-1} x$ equals ($x \in [-1, 1]$): (a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) π [1 mark]

Ans: (c) $\pi/2$. Standard complement identity.

Q4. $\tan^{-1}(-1)$ equals: (a) $\pi/4$ (b) $3\pi/4$ (c) $-\pi/4$ (d) $-3\pi/4$ [1 mark]

Ans: (c) $-\pi/4$. $\tan(-\pi/4) = -1$ and $-\pi/4 \in (-\pi/2, \pi/2)$.

Section B — Short Answer (2 marks each)

Q5. Find the principal value of $\cos^{-1}(-\sqrt{3}/2)$. [2 marks]

Ans: Let $\theta = \cos^{-1}(-\sqrt{3}/2)$. Then $\cos\theta = -\sqrt{3}/2$ and $\theta \in [0, \pi]$. $\cos(5\pi/6) = -\cos(\pi/6) = -\sqrt{3}/2$. Since $5\pi/6 \in [0, \pi]$, $\theta = 5\pi/6$.

Q6. Evaluate $\sin^{-1}(\sin(7\pi/6))$. [2 marks]

Ans: $7\pi/6 \notin [-\pi/2, \pi/2]$. $\sin(7\pi/6) = -\sin(\pi/6) = -1/2$. $\sin^{-1}(-1/2) = -\pi/6$ (since $-\pi/6 \in [-\pi/2, \pi/2]$). Answer: $-\pi/6$.

Q7. Find the principal value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$. [2 marks]

Ans: $\tan^{-1}(\sqrt{3}) = \pi/3$. $\sec^{-1}(-2) = \pi - \sec^{-1}(2) = \pi - \pi/3 = 2\pi/3$. Difference $= \pi/3 - 2\pi/3 = -\pi/3$.

Q8. If $\sin^{-1} x = \pi/5$ for some $x \in [-1, 1]$, find $\cos^{-1} x$. [2 marks]

Ans: Using $\sin^{-1} x + \cos^{-1} x = \pi/2$: $\cos^{-1} x = \pi/2 - \pi/5 = 5\pi/10 - 2\pi/10 = 3\pi/10$.

Section C — Long Answer (3 marks each)

Q9. Prove that $\sin^{-1}(1/\sqrt{5}) + \sin^{-1}(2/\sqrt{5}) = \pi/2$. [3 marks]

Ans: Let $A = \sin^{-1}(1/\sqrt{5})$, $B = \sin^{-1}(2/\sqrt{5})$. Then $\sin A = 1/\sqrt{5}$, $\cos A = \sqrt{(1 - 1/5)} = 2/\sqrt{5}$; $\sin B = 2/\sqrt{5}$, $\cos B = 1/\sqrt{5}$. $\sin(A + B) = \sin A \cos B + \cos A \sin B = (1/\sqrt{5})(1/\sqrt{5}) + (2/\sqrt{5})(2/\sqrt{5}) = 1/5 + 4/5 = 1$. So $A + B = \pi/2$ (the unique angle in $[0, \pi]$ whose sine is 1). ∎

Q10. Show that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$. [3 marks]

Ans: $\tan^{-1}(1) = \pi/4$. For $\tan^{-1}(2) + \tan^{-1}(3)$: $xy = 6 > 1$, both positive ⇒ sum $= \pi + \tan^{-1}((2+3)/(1-6)) = \pi + \tan^{-1}(-1) = \pi - \pi/4 = 3\pi/4$. Total $= \pi/4 + 3\pi/4 = \pi$. ∎

Q11. Express $2\tan^{-1}(1/3)$ in the form $\tan^{-1}(p/q)$ with p, q positive integers. [3 marks]

Ans: Use $2\tan^{-1} x = \tan^{-1}(2x/(1 - x^2))$, valid since $|1/3| < 1$. With $x = 1/3$: $2x = 2/3$, $1 - x^2 = 1 - 1/9 = 8/9$. Ratio $= (2/3) \div (8/9) = (2/3) \times (9/8) = 3/4$. Hence $2\tan^{-1}(1/3) = \tan^{-1}(3/4)$.

Q12. Prove that $\sin^{-1} x + \cos^{-1} x = \pi/2$ for all $x \in [-1, 1]$. [3 marks]

Ans: Let $\sin^{-1} x = \theta$. Then $\sin\theta = x$ and $\theta \in [-\pi/2, \pi/2]$. Now $\cos(\pi/2 - \theta) = \sin\theta = x$, and $\pi/2 - \theta \in [0, \pi]$ = range of $\cos^{-1}$. So $\cos^{-1} x = \pi/2 - \theta = \pi/2 - \sin^{-1} x$. Rearranging: $\sin^{-1} x + \cos^{-1} x = \pi/2$. ∎

Section D — Case-Based (5 marks total)

Q13. **A surveyor uses a clinometer to measure the angle of elevation to the top of a tower. The clinometer's display shows the value of $\tan^{-1}$(opposite/adjacent). For a particular reading, opposite/adjacent = 1/2. (i) Find the principal value of $\tan^{-1}(1/2) + \tan^{-1}(1/3)$. [2 marks] (ii) The surveyor moves and now records two readings; the sum of their angle-elevation values is observed to be $\pi/4$. If the first reading gives $\tan^{-1}(1/4)$, find the second $\tan^{-1}$ value. [2 marks]** [4 marks]

> **Ans:** (i) $xy = 1/6 < 1$. $\tan^{-1}(1/2) + \tan^{-1}(1/3) = \tan^{-1}((1/2 + 1/3)/(1 - 1/6)) = \tan^{-1}((5/6)/(5/6)) = \tan^{-1}(1) = \pi/4$. (ii) Let second reading be $\tan^{-1} y$. Then $\tan^{-1}(1/4) + \tan^{-1} y = \pi/4 \Rightarrow \tan^{-1} y = \pi/4 - \tan^{-1}(1/4) = \tan^{-1}(1) - \tan^{-1}(1/4) = \tan^{-1}((1 - 1/4)/(1 + 1/4)) = \tan^{-1}((3/4)/(5/4)) = \tan^{-1}(3/5)$. So $y = 3/5$.

Q14. **Find the principal value of $\sin^{-1}(\sin(13\pi/7))$.** [2 marks]

> **Ans:** $13\pi/7 \notin [-\pi/2, \pi/2]$. Note $13\pi/7 = 2\pi - \pi/7$, so $\sin(13\pi/7) = -\sin(\pi/7)$. Now $\sin^{-1}(-\sin(\pi/7)) = -\pi/7$ (since $\pi/7 \in [-\pi/2, \pi/2]$ and $\sin^{-1}(\sin(-\pi/7)) = -\pi/7$). Answer: $-\pi/7$.

★ TOPPER ANSWER TEMPLATES

5 TEMPLATES · MEMORISE THE FORMAT

★ TOPPER TEMPLATE — Find principal value of inverse trig at standard input (1-2 marks)

Every paper, 1-2 instances

Step 1 [0.5 mark]	**State principal range**	We know $\sin^{-1}$ has principal range $[-\pi/2, \pi/2]$.
Step 2 [1 mark]	**Identify angle whose sine equals input**	Let $\sin^{-1}(-1/2) = \theta \Rightarrow \sin\theta = -1/2$ and $\theta \in [-\pi/2, \pi/2]$.
Step 3 [0.5 mark]	**State answer with justification**	$\theta = -\pi/6$ since $\sin(-\pi/6) = -1/2$ and $-\pi/6 \in [-\pi/2, \pi/2]$. Hence principal value = $-\pi/6$.

COMMON LOSS OF MARKS:

- Forgetting to verify the angle lies in the principal range (gives $5\pi/6$ instead of $-\pi/6$ for $\sin^{-1}(-1/2)$)
- Writing answer in degrees without converting (always use radians in CBSE)
- Skipping the 'let θ = ...' step costs 0.5 marks

★ TOPPER TEMPLATE — Evaluate $\sin^{-1}(\sin x)$ or $\cos^{-1}(\cos x)$ where x is outside principal range (2-3 marks)

Once every 2 papers

Step 1 [0.5 mark]	**Check if x is in principal range**	Given $\cos^{-1}(\cos(7\pi/6))$. Principal range of $\cos^{-1}$ is $[0, \pi]$. Here $7\pi/6 \notin [0, \pi]$.
Step 2 [1 mark]	**Reduce x to equivalent angle in range**	$\cos(7\pi/6) = \cos(2\pi - 7\pi/6)$ is wrong; use $\cos(7\pi/6) = -\sqrt{3}/2$ directly. Find $\theta \in [0, \pi]$ with $\cos\theta = -\sqrt{3}/2$.
Step 3 [1 mark]	**State principal answer**	$\cos(5\pi/6) = -\sqrt{3}/2$ and $5\pi/6 \in [0, \pi]$, so $\cos^{-1}(\cos(7\pi/6)) = 5\pi/6$.
Step 4 [0.5 mark]	**Verify final lies in range**	$5\pi/6 \in [0, \pi]$ ✓. Final answer: $5\pi/6$.

COMMON LOSS OF MARKS:

- Writing $7\pi/6$ directly (assumes $\sin^{-1}(\sin x) = x$ for all x — wrong)
- Confusing sin and cos reduction formulae
- Not stating the principal range explicitly

★ TOPPER TEMPLATE — Prove identity such as $\tan^{-1} x + \tan^{-1} y = \tan^{-1}((x+y)/(1-xy))$ (3 marks)

Once every paper

Step 1 [0.5 mark]	**Substitute variables**	Let $\tan^{-1} x = A \Rightarrow \tan A = x$; $\tan^{-1} y = B \Rightarrow \tan B = y$.
Step 2 [1 mark]	**Apply sum formula for tan**	$\tan(A+B) = (\tan A + \tan B)/(1 - \tan A \tan B) = (x+y)/(1-xy)$.
Step 3 [1 mark]	**Take inverse and check range condition**	$A + B = \tan^{-1}((x+y)/(1-xy))$, valid when $xy < 1$ (so $A+B \in (-\pi/2, \pi/2)$).
Step 4 [0.5 mark]	**Conclude**	Hence $\tan^{-1} x + \tan^{-1} y = \tan^{-1}((x+y)/(1-xy))$ provided $xy < 1$.

COMMON LOSS OF MARKS:

- Omitting the condition $xy < 1$ (always −0.5)
- Algebra slip in $\tan(A+B)$ formula
- Not mentioning the case $xy > 1$ (add π or subtract π)

★ TOPPER TEMPLATE — Numerical: evaluate sum of multiple $\tan^{-1}$ values (3 marks)

Once every 2 papers

Step 1 [0.5 mark]	**Identify standard values**	$\tan^{-1}(1) = \pi/4$ (standard).
Step 2 [1.5 marks]	**Combine $\tan^{-1}(2) + \tan^{-1}(3)$ using formula with sign check**	Here $xy = 6 > 1$, both positive $\Rightarrow \tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}((2+3)/(1-6)) = \pi + \tan^{-1}(-1) = \pi - \pi/4 = 3\pi/4$.
Step 3 [1 mark]	**Add and box final answer**	$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi/4 + 3\pi/4 = \pi$.

COMMON LOSS OF MARKS:

- Forgetting $+\pi$ adjustment when $xy > 1$ — gives wrong answer 0 instead of π
- Sign error in $(1 - xy)$
- Not showing standard value derivations

★ TOPPER TEMPLATE — Use $2 \tan^{-1} x$ identity to simplify (3 marks)

Once every 2-3 papers

Step 1 [0.5 mark]	**Identify which identity applies**	We need $2 \tan^{-1} x = \tan^{-1}(2x/(1-x^2))$, valid for $\lvert x\rvert < 1$.
Step 2 [1 mark]	**Substitute given x**	For $x = 1/3$: $2x/(1-x^2) = (2/3)/(1 - 1/9) = (2/3)/(8/9) = (2/3) \cdot (9/8) = 3/4$.
Step 3 [1 mark]	**Check validity range**	$\lvert 1/3\rvert < 1$ ✓. So $2 \tan^{-1}(1/3) = \tan^{-1}(3/4)$.
Step 4 [0.5 mark]	**State result clearly**	Final: $2 \tan^{-1}(1/3) = \tan^{-1}(3/4)$.

COMMON LOSS OF MARKS:

- Not checking $\lvert x\rvert < 1$ condition
- Arithmetic slip in $(1 - x^2)$
- Confusing $2 \sin^{-1}$ (no such identity) with $2 \tan^{-1}$

MARKING SCHEME — GENERAL NOTES

- Award full marks for correct final answer with complete working (let $\theta = \ldots$ → range check → reduction → final).
- Deduct 0.5 marks per question if the principal range is not stated explicitly.
- In identity proofs, omitting the validity condition (e.g., $xy < 1$ for $\tan^{-1}$ sum) costs 0.5 marks.
- Answers in degrees are NOT accepted — radians only.
- Numerical/arithmetic slips in standard values ($\sin(\pi/6) = 1/2$ etc.) lose 1 mark but allow ECF (error carried forward) for subsequent steps.
- For $\sin^{-1}(\sin x)$ type problems, the student MUST reduce x first; writing the input as the answer without justification scores 0 even if the input happens to lie in range.
- Section D case-based: marks are awarded for setting up the $\tan^{-1}$ sum identity correctly even if the final arithmetic slips.