CHAPTER 2

Inverse Trigonometric Functions

CBSE · Mathematics · Class 12

WHAT THIS CHAPTER DOES

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's Mission

WHY THIS MATTERS

Why do inverse trig functions exist?

sin x maps every real number to [−1, 1] — but many angles share the same sine value ($\sin(\pi/6) = \sin(5\pi/6) = 1/2$). To invert, we must RESTRICT the domain so that sine becomes one-one. The restricted domain $[-\pi/2, \pi/2]$ is called the PRINCIPAL BRANCH. Within it, sin is one-one and onto [−1, 1]. The inverse, $\sin^{-1}$, takes a ratio in [−1, 1] back to a unique angle in $[-\pi/2, \pi/2]$.

TOPIC

Pre-flight Check

TOPIC

A

Part A — The 6 Principal Value Branches

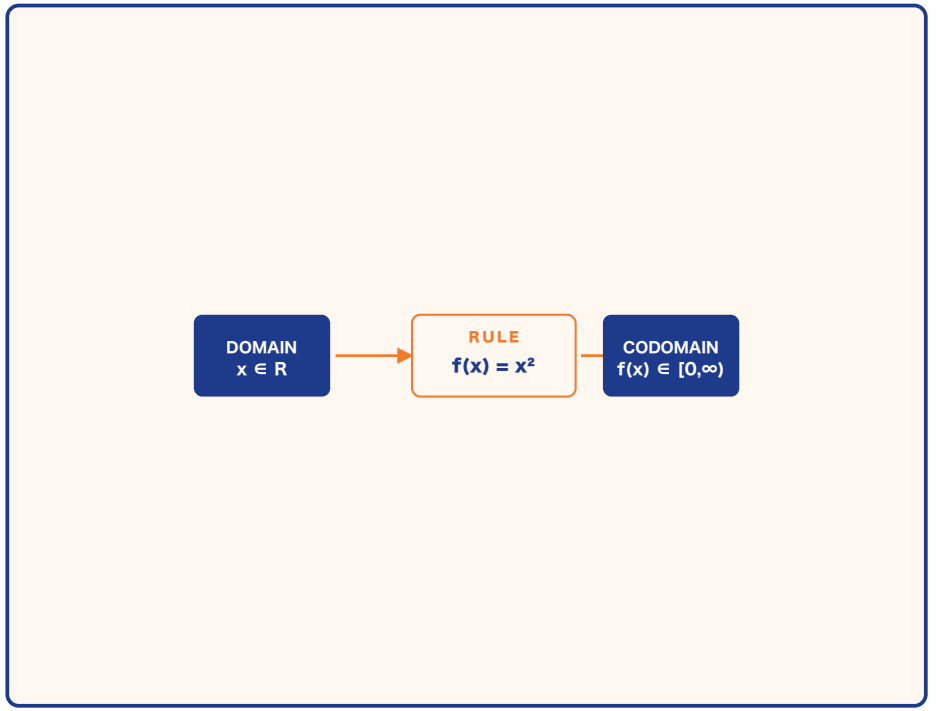
TOPIC

Principal Value Table — Memorise This

POINT 1	POINT 2	POINT 3	POINT 4

WORKED EXAMPLE

Worked Example 1 — Visualising $\sin^{-1}$ as an Arrow



TRY IT · SOLVE BEFORE YOU PEEK

Quick Test — 60 seconds

What is the principal value of $\cos^{-1}(-\sqrt{3}/2)$?

SOLUTION

TOPIC

Standard Inverse Trig Values — Reference Grid

WORKED EXAMPLE

Worked Example 2 — Reduction outside principal range

TOPIC

Core Properties — Negative Inputs & Complement Pairs

POINT 1	POINT 2	POINT 3	POINT 4

TOPIC

B

Part B — Key Identities

WORKED EXAMPLE

Worked Example 3 — Prove $\sin^{-1} x + \cos^{-1} x = \pi/2$

TRY IT · SOLVE BEFORE YOU PEEK

Quick Test — apply complement

If $\sin^{-1} x = \pi/3$, find $\cos^{-1} x$.

SOLUTION

TOPIC

Double-Angle Tan Identities

POINT 1	POINT 2	POINT 3	POINT 4

TOPPER TEMPLATE · MARK-BY-MARK

Topper Template — 3-mark Identity Proof

TOPIC

C

Part C — Sum Formulae in Action

WORKED EXAMPLE

Worked Example 4 — $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$

TOPIC

Self-Assessment — Can you do these without notes?

MARKS DISTRIBUTION

Marks Heatmap — Where the Marks Hide

PYQ PATTERNS

PYQ Patterns — What CBSE Asks (2018-2024)

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** $\sin^{-1}(\sin x) = x$ for every real x.

✓ **CORRECT** Only when $x \in [-\pi/2, \pi/2]$. Otherwise, reduce x to an equivalent angle within the principal range first.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** $\sin^{-1} x$ means $1/\sin x$ (reciprocal).

✓ **CORRECT** $\sin^{-1} x$ means the angle whose sine is x. The reciprocal of sin x is cosec $x = (\sin x)^{-1}$, written differently.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** $\cos^{-1}(-x) = -\cos^{-1} x$ (treating it like sine).

✓ **CORRECT** $\cos^{-1}(-x) = \pi - \cos^{-1} x$. Range of $\cos^{-1}$ is $[0, \pi]$, which is non-negative — it cannot return a negative value.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** $\tan^{-1} x + \tan^{-1} y = \tan^{-1}((x+y)/(1-xy))$ always.

✓ **CORRECT** Valid only when $xy < 1$. If $xy > 1$ and $x, y > 0$, add π; if $xy > 1$ and $x, y < 0$, subtract π.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** Domain of $\sec^{-1}$ is all reals.

✓ **CORRECT** Domain of $\sec^{-1}$ is $|x| \geqslant 1$, i.e., $(-\infty, -1] \cup [1, \infty)$. $\sec^{-1}(1/2)$ does not exist.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** Principal value of $\cos^{-1}(-1)$ is undefined or $-\pi$.

✓ **CORRECT** $\cos^{-1}(-1) = \pi$. Range $[0, \pi]$ is inclusive of both ends.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** $2 \sin^{-1} x = \sin^{-1}(2x)$ always.

✓ **CORRECT** There is NO such identity. The correct identity is $2 \tan^{-1} x = \tan^{-1}(2x/(1-x^2))$ for $|x| < 1$, valid for $\tan^{-1}$ only.

TOPPER TEMPLATE · MARK-BY-MARK

Find principal value of inverse trig at standard input (1-2 marks)

1	**STATE PRINCIPAL RANGE** 0.5 m	We know $\sin^{-1}$ has principal range $[-\pi/2, \pi/2]$.
2	**IDENTIFY ANGLE WHOSE SINE EQUALS INPUT** 1 m	Let $\sin^{-1}(-1/2) = \theta \Rightarrow \sin\theta = -1/2$ and $\theta \in [-\pi/2, \pi/2]$.
3	**STATE ANSWER WITH JUSTIFICATION** 0.5 m	$\theta = -\pi/6$ since $\sin(-\pi/6) = -1/2$ and $-\pi/6 \in [-\pi/2, \pi/2]$. Hence principal value $= -\pi/6$.

TOPPER TEMPLATE · MARK-BY-MARK

Evaluate $\sin^{-1}(\sin x)$ or $\cos^{-1}(\cos x)$ where x is outside principal range (2-3 marks)

1	**CHECK IF X IS IN PRINCIPAL RANGE** 0.5 m	Given $\cos^{-1}(\cos(7\pi/6))$. Principal range of $\cos^{-1}$ is $[0, \pi]$. Here $7\pi/6 \notin [0, \pi]$.
2	**REDUCE X TO EQUIVALENT ANGLE IN RANGE** 1 m	$\cos(7\pi/6) = \cos(2\pi - 7\pi/6)$ is wrong; use $\cos(7\pi/6) = -\sqrt{3}/2$ directly. Find $\theta \in [0, \pi]$ with $\cos\theta = -\sqrt{3}/2$.
3	**STATE PRINCIPAL ANSWER** 1 m	$\cos(5\pi/6) = -\sqrt{3}/2$ and $5\pi/6 \in [0, \pi]$, so $\cos^{-1}(\cos(7\pi/6)) = 5\pi/6$.
4	**VERIFY FINAL LIES IN RANGE** 0.5 m	$5\pi/6 \in [0, \pi]$ ✓. Final answer: $5\pi/6$.

TOPPER TEMPLATE · MARK-BY-MARK

Prove identity such as $\tan^{-1} x + \tan^{-1} y = \tan^{-1}((x+y)/(1-xy))$ (3 marks)

1	**SUBSTITUTE VARIABLES** 0.5 m	Let $\tan^{-1} x = A \Rightarrow \tan A = x$; $\tan^{-1} y = B \Rightarrow \tan B = y$.
2	**APPLY SUM FORMULA FOR TAN** 1 m	$\tan(A+B) = (\tan A + \tan B)/(1 - \tan A \tan B) = (x+y)/(1-xy)$.
3	**TAKE INVERSE AND CHECK RANGE CONDITION** 1 m	$A + B = \tan^{-1}((x+y)/(1-xy))$, valid when $xy < 1$ (so $A+B \in (-\pi/2, \pi/2)$).
4	**CONCLUDE** 0.5 m	Hence $\tan^{-1} x + \tan^{-1} y = \tan^{-1}((x+y)/(1-xy))$ provided $xy < 1$.

PYQ PATTERNS

Top PYQ patterns to drill

#1	Principal value of $\sin^{-1}/\cos^{-1}/\tan^{-1}$ at standard input (2 marks)	95%
#2	$\sin^{-1}(\sin x)$ or $\cos^{-1}(\cos x)$ where x outside principal range (2 marks)	70%
#3	Prove identity $\sin^{-1} + \cos^{-1} = \pi/2$ etc. (3 marks)	55%
#4	Sum $\tan^{-1} x + \tan^{-1} y$ with $xy>1$ case (3 marks)	40%
#5	$2 \tan^{-1} x$ identity simplification (3 marks)	30%

RECAP · MEMORISE THESE

One-Slide Summary

WHAT'S NEXT

Next Class — Chapter 3: Matrices

Your 90-Minute Mastery Plan

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