ANSWER KEY & MARKING SCHEME · CBSE CLASS 12

Matrices

Mathematics · Chapter 3 · Use this with the Board Paper · Companion to Quick Drill

HOW TO USE

Attempt the Board Paper first (closed-book, full time). Then come here. For 2-mark+ questions, compare your answer to the model. For 3-4 mark questions, also consult the **Topper Templates** below — these show the exact step-by-step structure that scores full marks per CBSE marking-scheme conventions.

MODEL ANSWERS · BOARD PAPER

Section A — MCQ (1 mark each)

Q1. A matrix with 12 elements can have how many possible orders? (a) 3 (b) 4 (c) 6 (d) 12 [1 mark]

Ans: (c) 6. mn = 12 gives (1,12),(12,1),(2,6),(6,2),(3,4),(4,3).

Q2. For matrices A and B, (AB)^T equals: (a) A^T B^T (b) B^T A^T (c) AB (d) BA [1 mark]

Ans: (b) B^T A^T. The transpose of a product reverses the order.

Q3. Every diagonal element of a skew-symmetric matrix is: (a) 1 (b) 0 (c) −1 (d) arbitrary [1 mark]

Ans: (b) 0. A^T = −A forces a_ii = −a_ii ⇒ a_ii = 0.

Q4. If A is 2×3 and B is 3×4, the order of AB is: (a) 2×4 (b) 3×3 (c) 4×2 (d) not defined [1 mark]

Ans: (a) 2×4. Inner dimensions 3 = 3 match; order is (rows of A)×(cols of B).

Section B — Short Answer (2 marks each)

Q5. If [[x + y, 2], [5, x − y]] = [[6, 2], [5, 2]], find x and y. [2 marks]

Ans: Equate entries: x + y = 6, x − y = 2. Adding gives 2x = 8 ⇒ x = 4; then y = 2.

Q6. If A = [[1, 2], [3, 4]] and B = [[0, 1], [1, 0]], compute AB. [2 marks]

Ans: Row 1: [1·0+2·1, 1·1+2·0] = [2, 1]. Row 2: [3·0+4·1, 3·1+4·0] = [4, 3]. So AB = [[2, 1], [4, 3]].

Q7. Show that the matrix A = [[0, 5], [−5, 0]] is skew-symmetric. [2 marks]

Ans: A^T = [[0, −5], [5, 0]] = −[[0, 5], [−5, 0]] = −A, and both diagonal entries are 0. Hence A is skew-symmetric.

Q8. If 3A = [[6, 9], [3, 12]], find the matrix A. [2 marks]

Ans: Scalar multiplication is entrywise, so A = (1/3)(3A) = [[2, 3], [1, 4]].

Section C — Long Answer (3 marks each)

Q9. Express the matrix A = [[3, 5], [1, −1]] as the sum of a symmetric and a skew-symmetric matrix. [3 marks]

Ans: A^T = [[3, 1], [5, −1]]. Symmetric part P = ½(A + A^T) = ½[[6, 6], [6, −2]] = [[3, 3], [3, −1]]. Skew part Q = ½(A − A^T) = ½[[0, 4], [−4, 0]] = [[0, 2], [−2, 0]]. Then A = P + Q with P^T = P and Q^T = −Q. ∎

Q10. For A = [[1, 2], [0, 1]] and B = [[1, 0], [3, 1]], verify that (AB)^T = B^T A^T. [3 marks]

Ans: AB = [[1·1+2·3, 1·0+2·1], [0·1+1·3, 0·0+1·1]] = [[7, 2], [3, 1]], so (AB)^T = [[7, 3], [2, 1]]. Now A^T = [[1, 0], [2, 1]], B^T = [[1, 3], [0, 1]], and B^T A^T = [[1·1+3·2, 1·0+3·1], [0·1+1·2, 0·0+1·1]] = [[7, 3], [2, 1]]. Both equal, so (AB)^T = B^T A^T. ∎

Q11. Show that for any square matrix A, the matrix A + A^T is symmetric and A − A^T is skew-symmetric. [3 marks]

Ans: Let P = A + A^T. Then P^T = (A + A^T)^T = A^T + (A^T)^T = A^T + A = P, so P is symmetric. Let Q = A − A^T. Then Q^T = A^T − (A^T)^T = A^T − A = −(A − A^T) = −Q, so Q is skew-symmetric. ∎

Q12. **If $A = [[2, 0], [0, 3]]$ and $B = [[1, 1], [0, 1]]$, compute AB and BA and state whether AB = BA.** **[3 marks]**

> **Ans:** $AB = [[2\cdot1+0\cdot0,\ 2\cdot1+0\cdot1], [0\cdot1+3\cdot0,\ 0\cdot1+3\cdot1]] = [[2, 2], [0, 3]]$. $BA = [[1\cdot2+1\cdot0,\ 1\cdot0+1\cdot3], [0\cdot2+1\cdot0,\ 0\cdot0+1\cdot3]] = [[2, 3], [0, 3]]$. Since $[[2, 2], [0, 3]] \neq [[2, 3], [0, 3]]$, $AB \neq BA$.

Section D — Case-Based / Inverse (6 marks total)

Q13. **Using elementary ROW operations, find the inverse of $A = [[2, 3], [1, 2]]$, and verify your answer by computing $A\cdot A^{-1}$.** **[5 marks]**

> **Ans:** $|A| = 4 - 3 = 1 \neq 0$, so A is invertible. Write $A = IA$. $R_1 \leftrightarrow R_2$: $[[1, 2], [2, 3]] = [[0, 1], [1, 0]]A$. $R_2 \to R_2 - 2R_1$: $[[1, 2], [0, -1]] = [[0, 1], [1, -2]]A$. $R_2 \to (-1)R_2$: $[[1, 2], [0, 1]] = [[0, 1], [-1, 2]]A$. $R_1 \to R_1 - 2R_2$: $[[1, 0], [0, 1]] = [[2, -3], [-1, 2]]A$. Hence $A^{-1} = [[2, -3], [-1, 2]]$. Verify: $A\cdot A^{-1} = [[2\cdot2+3\cdot(-1),\ 2\cdot(-3)+3\cdot2], [1\cdot2+2\cdot(-1),\ 1\cdot(-3)+2\cdot2]] = [[1, 0], [0, 1]] = I$. ∎

Q14. **A square matrix A satisfies $A^2 = A$ and is invertible. Using the inverse, show that A must equal the identity matrix I.** **[1 mark]**

> **Ans:** Since A is invertible, A^{-1} exists. Multiply $A^2 = A$ on the left by A^{-1}: $A^{-1}A^2 = A^{-1}A \Rightarrow (A^{-1}A)A = I \Rightarrow IA = I \Rightarrow A = I$.

★ TOPPER ANSWER TEMPLATES

5 TEMPLATES · MEMORISE THE FORMAT

★ TOPPER TEMPLATE — State order of a matrix / list possible orders given number of elements (1-2 marks)

Every paper, 1-2 instances

Step 1 **[0.5 mark]**	**Recall order means rows × columns**	An m×n matrix has m rows and n columns, with mn total entries.
Step 2 **[1 mark]**	**List all factor pairs of the entry count**	For 12 elements, $mn = 12 \Rightarrow (m, n) \in \{(1,12), (12,1), (2,6), (6,2), (3,4), (4,3)\}$.
Step 3 **[0.5 mark]**	**Count and state**	There are 6 possible orders.

COMMON LOSS OF MARKS:

- Listing only 'distinct shapes' (3) instead of ordered pairs (6) — order m×n and n×m are different
- Forgetting the 1×12 and 12×1 trivial pairs
- Writing order as n×m by mistake

★ TOPPER TEMPLATE — Find unknown entries by equality of matrices (1-2 marks)

Once every paper

Step 1 **[0.5 mark]**	**State the equality condition**	Two matrices are equal iff they have the same order AND corresponding entries are equal.
Step 2 **[1 mark]**	**Write the entrywise equations**	From $[[x+y, 2],[5, x-y]] = [[6, 2],[5, 2]]$: $x + y = 6$ and $x - y = 2$.
Step 3 **[0.5 mark]**	**Solve and state**	Adding: $2x = 8 \Rightarrow x = 4$; substituting, $y = 2$.

COMMON LOSS OF MARKS:

- Matching entries in the wrong positions
- Solving only one equation and stopping
- Not checking the order is identical before equating

★ TOPPER TEMPLATE — Verify transpose property such as (AB)^T = B^T A^T (3 marks)

Once every 1-2 papers

Step 1 [0.5 mark]	**Compute the product AB**	With A and B given, multiply rows of A by columns of B to obtain AB.
Step 2 [1 mark]	**Take the transpose of the product**	(AB)^T is AB with rows and columns interchanged.
Step 3 [1 mark]	**Compute B^T A^T separately**	Find B^T and A^T, then multiply in the reversed order B^T A^T.
Step 4 [0.5 mark]	**Compare and conclude**	Both matrices are identical, hence (AB)^T = B^T A^T is verified. ∎

COMMON LOSS OF MARKS:

- Computing A^T B^T (same order) instead of the reversed B^T A^T
- Arithmetic slip in the product AB
- Not stating the conclusion explicitly

★ TOPPER TEMPLATE — Express a square matrix as sum of symmetric and skew-symmetric matrices (3-5 marks)

Once every 1-2 papers

Step 1 [1 mark]	**Write A^T**	For A = [[3, 5],[1, −1]], A^T = [[3, 1],[5, −1]].
Step 2 [1.5 marks]	**Form the symmetric part P = (A + A^T)/2**	P = ½[[6, 6],[6, −2]] = [[3, 3],[3, −1]], and P^T = P.
Step 3 [1.5 marks]	**Form the skew-symmetric part Q = (A − A^T)/2**	Q = ½[[0, 4],[−4, 0]] = [[0, 2],[−2, 0]], and Q^T = −Q.
Step 4 [1 mark]	**State A = P + Q and verify**	P + Q = [[3, 5],[1, −1]] = A ✓. Hence A is written as symmetric P plus skew-symmetric Q.

COMMON LOSS OF MARKS:

- Forgetting the factor ½ in P and Q
- Sign error in A − A^T for the skew part
- Not verifying P^T = P and Q^T = −Q

★ TOPPER TEMPLATE — Find inverse of a 2×2 or 3×3 matrix by elementary row operations (5 marks)

Once every 2 papers

Step 1 [1 mark]	**Set up A = IA**	Write A = IA: place A on the left, I on the right, agreeing to apply only ROW operations.
Step 2 [2 marks]	**Reduce the left side to I using row operations**	Apply elementary row operations to make the left matrix the identity, mirroring every step on the right matrix.
Step 3 [1.5 marks]	**Read off the inverse**	When the left becomes I, the right matrix is A^{-1}.
Step 4 [0.5 mark]	**Verify $A \cdot A^{-1}$ = I**	Multiply A by the obtained inverse to confirm the product is the identity.

COMMON LOSS OF MARKS:

- Mixing row and column operations (instant loss of method marks)
- Not applying the SAME operation to the right side at every step
- Stopping before the left side is fully the identity

MARKING SCHEME — GENERAL NOTES

- Award full marks for the correct final matrix with complete working and correct order/brackets.
- For multiplication, deduct 0.5 marks per question if compatibility of orders is not checked before multiplying.
- In transpose proofs, using (AB)^T = A^T B^T instead of the reversed B^T A^T scores 0 for that step.
- For symmetric/skew-symmetric decomposition, omitting the factor ½ in P or Q loses 1 mark.
- For inverse by elementary operations, mixing row and column operations loses all method marks; only ROW operations are accepted for the A = IA setup.
- Arithmetic slips in a product entry lose 1 mark but allow ECF (error carried forward) for subsequent steps.
- Section D inverse: marks are awarded step-wise for each correct elementary operation, even if a later step slips.