CHAPTER 1

Number Systems

CBSE Class 9 · Mathematics · Chapter 1

CBSE · Mathematics · Class 9

WHAT THIS CHAPTER DOES

A Tell a rational number from an irrational one — using its decimal expansion or its p/q form.

B Place any real number, including $\sqrt{5}$ and $\sqrt{9.3}$, exactly on the number line.

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's mission

1. Tell a rational number from an irrational one — using its decimal expansion or its p/q form.
2. Place any real number, including $\sqrt{5}$ and $\sqrt{9.3}$, exactly on the number line.
3. Rationalise the denominator of any $1/(a + b\sqrt{x})$ expression using the conjugate.
4. Simplify any expression using the laws of exponents for real bases — and score 8/10 on Unit I.

WHY THIS MATTERS

Why this chapter matters

1. Unit I (Number Systems) is worth 10 marks in the Class 9 blueprint — and this chapter is its core.
2. Every later chapter — Polynomials, Coordinate Geometry, Surd algebra in Class 10 — ASSUMES you can handle irrationals and exponents fluently.
3. It is the first time numbers stop being 'countable fractions' and become a continuous, gap-free line — the idea underlying all of calculus later.

TOPIC

A

Fundamentals — the family of numbers

TOPIC

The nested number families · $N \subset W \subset Z \subset Q \subset R$

NATURALS, WHOLES, INTEGERS

The counting numbers 1, 2, 3, … are the natural numbers (N). Adding zero gives the whole numbers (W = 0, 1, 2, …). Adding the negatives gives the integers (Z = …, −2, −1, 0, 1, 2, …). Each set sits INSIDE the next — every natural number is a whole number, every whole number is an

RATIONAL NUMBERS (Q)

A rational number is any number that can be written as p/q where p and q are INTEGERS and q ≠ 0. This includes every integer (5 = 5/1), every fraction (3/7), every terminating decimal (0.25 = 1/4), and every recurring decimal (0.bar(3) = 1/3). The two conditions matter: q must not be zero (division by

IRRATIONAL NUMBERS

An irrational number CANNOT be written as p/q for any integers p, q. Its decimal expansion runs on forever WITHOUT ever settling into a repeating block. The classic examples are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ (square roots of non-perfect-squares), the golden ratio, and $\pi \approx$ 3.14159… Beware: 22/7 is only an

REAL NUMBERS (R)

Putting the rationals and irrationals together gives the REAL numbers — and the deep result of this chapter is that the real numbers fill the number line COMPLETELY, with no gaps. Every point on the line is exactly one real number, and every real number is exactly one point. This 'one-to-one correspondence'

THEOREM · LOAD-BEARING RESULT

Rationals are dense — and so are irrationals

> " Between any two distinct real numbers there exist infinitely many rational numbers AND infinitely many irrational numbers.

STATEMENT

Given $a < b$, the number $(a+b)/2$ is a rational lying strictly between them whenever a, b are rational; repeating the averaging yields infinitely many. Likewise one can always insert a non-

WHY THIS MATTERS

- It tells you the number line has no smallest gap
- This is why 'find 3 rational numbers between 1/3 and 1/2' always has an answer — and infinitely many of them.

WATCH OUT FOR

NOTE To find a rational between two numbers, averaging is safest. To find an IRRATIONAL between, say, 0.5 and 0.6, write a non-repeating decimal such as 0.51010010001000001... — do NOT just write another fraction.

TOPIC

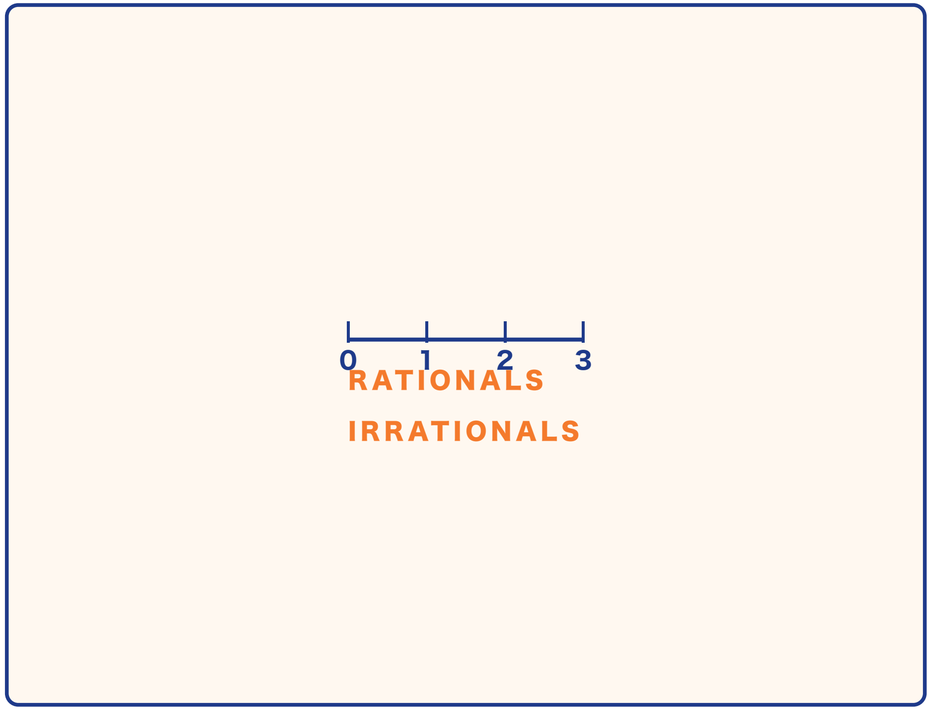
B

Locating irrationals on the number line

WORKED EXAMPLE

Represent $\sqrt{2}$ and $\sqrt{3}$ on the number line

1. For $\sqrt{2}$: mark O at 0 and A at 1 on the number line. Draw AB ⊥ to the line with AB = 1 unit.
2. By Pythagoras, $OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$. So OB has length $\sqrt{2}$.
3. With O as centre and radius OB, draw an arc cutting the number line at P. Then $OP = \sqrt{2}$ — label P as $\sqrt{2}$.
4. For $\sqrt{3}$: now take $OB = \sqrt{2}$ as the base, erect a new 1-unit perpendicular BC; $OC = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$. This 'square-root spiral' continues for $\sqrt{4}$, $\sqrt{5}$, …



TOPIC

Successive magnification · trapping a decimal point

THE IDEA

Successive magnification is how we locate a number with a long decimal — like 3.765 — on the number line precisely. We treat the number line like a microscope: first we see that 3.765 lies between 3 and 4. We then 'zoom in' on the segment from 3 to 4, dividing it into 10 equal parts to see it lies

WHY IT WORKS FOR REALS

Successive magnification proves visually that EVERY real number, however long its decimal, occupies a single exact point. Because we can always zoom one more level and split into ten, there is no decimal we cannot eventually trap — even a non-terminating one like π, where each new zoom reveals the

HOW TO DRAW IT

Draw the first number line normally and circle the unit interval containing the number. Beside it, draw a magnified copy of just that interval, ten times longer, marked in tenths, and circle the relevant tenth. Draw a third line magnifying that tenth into hundredths, and so on. Connect each circled

TERMINATING VS RECURRING

Successive magnification reaches an END for a terminating decimal (3.765 needs only three zooms), but for a recurring decimal like 4.bar(26) = 4.262626... the zooming pattern REPEATS forever in a predictable cycle — which is itself the visual signature of a rational

TOPIC

C

Decimal expansions — the rational/ irrational test

THEOREM · LOAD-BEARING RESULT

The decimal-expansion test

> A real number is RATIONAL if and only if its decimal expansion is either terminating OR non-terminating and recurring (repeating). It is IRRATIONAL if and only if its decimal expansion is non-terminating and non-recurring.

STATEMENT

For p/q in lowest terms, the expansion terminates exactly when q has only 2 and 5 as prime factors; otherwise it recurs with a repeating block of length at most (q − 1). A non-repeating, non-ending

WHY THIS MATTERS

- This single test lets you classify any number from its decimal alone, without finding p and q
- It is the most-tested idea in the chapter.

WATCH OUT FOR

NOTE Do NOT conclude a number is irrational just because it 'looks long'. 0.142857142857... is non-terminating but RECURRING (block '142857' = 1/7) — it is rational. Only NON-recurring non-terminating decimals are irrational.

WORKED EXAMPLE

Express 0.bar(3) and 1.2bar(7) in p/q form

1. Let x = 0.3333... One digit repeats, so multiply by 10: 10x = 3.3333...
2. Subtract: 10x − x = 3.3333... − 0.3333... ⇒ 9x = 3 ⇒ x = 3/9 = 1/3.
3. For 1.2bar(7) = 1.27777...: let y = 1.2777...; ×10 ⇒ 10y = 12.777...; ×100 ⇒ 100y = 127.777...
4. Subtract: 100y − 10y = 127.777... − 12.777... ⇒ 90y = 115 ⇒ y = 115/90 = 23/18.

TRY IT · SOLVE BEFORE YOU PEEK

Classify each as rational or irrational with a one-line reason: (a) 0.bar(45) (b) √11 (c) 3.141141114... (d) √36.

SOLUTION

ANSWER (a) Rational — recurring decimal (= 45/99 = 5/11). (b) Irrational — √ of a non-perfect square. (c) Irrational — non-terminating, non-recurring (the gaps of 1s grow). (d) Rational — √36 = 6, an integer.

TOPIC

D

Operations on real numbers

TOPIC

Adding and multiplying surds

LIKE VS UNLIKE SURDS

Surds (square-root expressions) add and subtract only when they are LIKE — same number under the root. So $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$, treating $\sqrt{2}$ like a common 'unit', exactly as $3x + 5x = 8x$. But $\sqrt{2} + \sqrt{3}$ cannot be combined into a single surd — it stays $\sqrt{2} + \sqrt{3}$. A very common error is writing $\sqrt{2} + \sqrt{3} = \sqrt{5}$

MULTIPLYING AND DIVIDING

For multiplication and division the roots DO combine: $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\sqrt{a} \div \sqrt{b} = \sqrt{a/b}$, provided $a, b \geqslant 0$. So $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$ (a rational result from two irrationals!), and $\sqrt{15} \div \sqrt{3} = \sqrt{5}$. These identities let you simplify before computing. They also generate the chapter's

CLOSURE FACTS TO MEMORISE

Rational ± rational = rational, and rational × rational = rational — the rationals are 'closed'. But irrationals are NOT closed: irrational ± irrational and irrational × irrational may be either. The reliable rules CBSE tests are: (rational ≠ 0) + irrational = irrational; (rational ≠ 0) × irrational

USEFUL IDENTITIES

Three algebraic identities do most of the heavy lifting with surds: $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$; $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$; and $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$. The first is the engine of rationalisation (it clears the root). The squaring identities appear when a question asks you to find $(3 + \sqrt{5})^2$ or to simplify $(\sqrt{7} - \sqrt{3})^2$

TOPIC

E

Rationalising the denominator

WORKED EXAMPLE

Rationalise $1/(3 + \sqrt{2})$ and $1/(\sqrt{7} - \sqrt{6})$

1. For $1/(3+\sqrt{2})$: the conjugate of $(3 + \sqrt{2})$ is $(3 - \sqrt{2})$. Multiply numerator and denominator by it.
2. $1/(3+\sqrt{2}) \times (3-\sqrt{2})/(3-\sqrt{2}) = (3-\sqrt{2})/((3)^2-(\sqrt{2})^2) = (3-\sqrt{2})/(9-2) = (3-\sqrt{2})/7$.
3. For $1/(\sqrt{7}-\sqrt{6})$: conjugate is $(\sqrt{7} + \sqrt{6})$. Multiply: $(\sqrt{7}+\sqrt{6})/((\sqrt{7})^2-(\sqrt{6})^2) = (\sqrt{7}+\sqrt{6})/(7-6)$.
4. Denominator = 1, so $1/(\sqrt{7}-\sqrt{6}) = \sqrt{7} + \sqrt{6}$. The surd has left the denominator — the goal of rationalisation.

TOPIC

Why and how we rationalise

WHAT RATIONALISING MEANS

Rationalising the denominator means rewriting a fraction so that NO square root (surd) remains in the denominator — the bottom becomes a plain rational number. We never change the VALUE of the fraction, only its appearance, by multiplying top and

SINGLE-SURD DENOMINATORS

When the denominator is a single surd like √5, just multiply numerator and denominator by that same surd: 1/√5 × √5/√5 = √5/5. The denominator √5 × √5 = 5 becomes rational. This is the simplest case and a guaranteed easy mark — never leave a lone √ in the denominator of a

TWO-TERM (BINOMIAL) DENOMINATORS

When the denominator is a sum or difference like (a + b√x), multiply by its CONJUGATE (a – b√x) — same terms, opposite middle sign. The identity (a+b)(a–b) = $a^2 - b^2$ then kills the cross terms and the surd disappears: (3+√2)(3–√2) = 9 – 2 = 7. The single most common mistake is multiplying by

CONJUGATE SIGN RULE

The conjugate is built by flipping the sign of the surd term and keeping everything else identical: conjugate of (5 + 2√3) is (5 – 2√3); conjugate of (√7 – √6) is (√7 + √6). After multiplying, simplify the rational denominator and reduce the fraction. If the question further asks for the nature of the result, remember the

TOPIC

F

Laws of exponents for real bases

THEOREM · LOAD-BEARING RESULT

Exponent laws extended to rational powers

> For a real base $a > 0$ and rational exponents p and q: $a^p \cdot aq = a^{p+}q$; $a^p \div aq = a^{p-}q$; $(a^p)q = a^pq$; $a^p \cdot b^p = (ab)^p$. Also $a^0 = 1$, $a^{-p} = 1/a^p$, and $a\^(1/n) = {}^n\sqrt{a}$.

STATEMENT

These are the same laws met in Class 8 for integer powers, now valid for rational exponents and positive real bases. In particular $a\^(m/n) = ({}^n\sqrt{a})^m = {}^n\sqrt{(a^m)}$, which is how a fractional power is

WHY THIS MATTERS

- They let you simplify root expressions algebraically instead of evaluating messy decimals — e.g
- $7\^(1/2) \cdot 7\^(1/3) = 7\^(5/6)$ in one line.

WATCH OUT FOR

NOTE ADD exponents when MULTIPLYING the same base ($a^p{\cdot}aq = a^{p+}q$); MULTIPLY exponents only when raising a power to a power ($(a^p)q = a^pq$). Confusing the two is the top error. Also the base a must be POSITIVE for these to hold.

WORKED EXAMPLE

Simplify 2^(2/3) · 2^(1/5) and (64)^(1/2) · (64)^(1/3)

1. 2^(2/3) · 2^(1/5): same base, MULTIPLYING ⇒ add exponents. 2/3 + 1/5 = 10/15 + 3/15 = 13/15.
2. So 2^(2/3) · 2^(1/5) = 2^(13/15).
3. (64)^(1/2) · (64)^(1/3): add exponents. 1/2 + 1/3 = 5/6, so = 64^(5/6).
4. Since 64 = 2^6, 64^(5/6) = (2^6)^(5/6) = 2^(6 × 5/6) = 2^5 = 32.

TRY IT · SOLVE BEFORE YOU PEEK

Simplify and give the simplest form: (a) $3^{(1/2)} \cdot 3^{(1/4)}$ (b) $(5^{(2/3)})^{(3)}$ (c) $7^{(1/2)} \div 7^{(1/3)}$.

SOLUTION

ANSWER (a) $3^{(1/2+1/4)} = 3^{(3/4)}$. (b) $5^{(2/3 \times 3)} = 5^2 = 25$. (c) $7^{(1/2-1/3)} = 7^{(1/6)}$.

TOPIC

Decimal expansion vs irrationality

TRAP → TRUTH

× **MISTAKE** Every non-terminating decimal is irrational.

✓ **CORRECT** Only non-terminating NON-RECURRING decimals are irrational. A non-terminating but RECURRING (repeating) decimal such as 0.333... = 1/3 or 0.142857142857... = 1/7 is perfectly rational. The test is: does the pattern repeat? If yes → rational; if it never repeats → irrational.

TOPIC

Sum / product of irrationals

TRAP → TRUTH

× **MISTAKE** The sum or product of two irrational numbers is always irrational.

✓ **CORRECT** NOT always. $\sqrt{2} \times \sqrt{2} = 2$ (rational); $(3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$ (rational); $\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$ (rational). It is only GUARANTEED irrational when you add/multiply a non-zero rational with an irrational. Treat each case on its own evidence.

TOPIC

Rationalising the denominator

TRAP → TRUTH

× **MISTAKE** To rationalise $1/(3+\sqrt{2})$, multiply top and bottom by $\sqrt{2}$.

✓ **CORRECT** Multiply by the CONJUGATE, $(3-\sqrt{2})$, not by $\sqrt{2}$. Using the identity $(a+b)(a-b)=a^2-b^2$, the denominator becomes $3^2-(\sqrt{2})^2 = 9-2 = 7$, which is rational. Multiplying by $\sqrt{2}$ leaves a $\sqrt{2}$ term in the denominator — the surd is not removed.

TOPIC

Square root of a negative

TRAP → TRUTH

× **MISTAKE** $\sqrt{(-4)} = -2$.

✓ **CORRECT** In Class 9 (real numbers) the square root of a NEGATIVE number does not exist on the real number line — there is no real number whose square is negative, because any real number squared is $\geqslant 0$. $(-2)^2 = +4$, not -4. $\sqrt{(-4)}$ is only defined later, in the complex-number system.

TOPIC

Locating $\sqrt{x}$ on the number line

TRAP → TRUTH

× **MISTAKE** To mark $\sqrt{5}$, just guess a point a little past 2.2 by eye.

✓ **CORRECT** Use the Pythagoras construction. To get $\sqrt{5}$: on the number line take OA = 2 units, draw AB = 1 unit perpendicular to it; then $OB = \sqrt{(2^2+1^2)} = \sqrt{5}$. With O as centre and radius OB, draw an arc cutting the number line — the cut point is $\sqrt{5}$ exactly, not by guessing.

TOPIC

Laws of exponents with rational powers

TRAP → TRUTH

× **MISTAKE** $a^{(1/2)} \times a^{(1/3)} = a^{(1/6)}$.

✓ **CORRECT** When MULTIPLYING powers of the same base you ADD the exponents: $a^{(1/2)} \times a^{(1/3)} = a^{(1/2 + 1/3)} = a^{(5/6)}$. You only multiply the exponents when raising a power to a power: $(a^{(1/2)})^{(1/3)} = a^{(1/6)}$. Mixing the two rules is the single most common exponent error.

TOPIC

p/q definition of rational

TRAP → TRUTH

× **MISTAKE** A rational number is any number that can be written as a fraction p/q.

✓ **CORRECT** Almost — but q must be NON-ZERO and both p, q must be INTEGERS. p/q with q = 0 is undefined, and 'π written as $\pi/1$' does not make π rational because π is not an integer. The exact definition: a number is rational if it can be written p/q where p and q are integers and $q \neq 0$.

TOPPER TEMPLATE · MARK-BY-MARK

2-3 mark question: 'Represent √x (e.g. √5 or √9.3) on the number line.'

1 — SET UP THE RIGHT TRIANGLE (OR SEMICIRCLE FOR √9.3) — 1 m

For a perfect-of-sum like √5: mark O at 0 and A at 2 on the number line (since $5 = 2^2 + 1^2$). Draw AB ⊥ to the line with AB = 1 unit. State clearly: 'OA = 2, AB = 1.' For √9.3 use the semicircle method: mark distance 9.3 then 1 unit further, draw a semicircle on the 10.3-unit diameter.

2 — APPLY PYTHAGORAS / THE GEOMETRIC MEAN — 1 m

By Pythagoras, $OB = \sqrt{OA^2 + AB^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$. Show this line of working — the examiner wants to see WHY OB equals √5, not just a labelled figure.

3 — TRANSFER THE LENGTH WITH A COMPASS ARC — 1 m

With O (or the appropriate centre) as centre and radius OB, draw an arc that cuts the number line at point P. Then OP = √5. Label P as √5 on the number line. A neat compass arc is mandatory for the final mark.

TOPPER TEMPLATE · MARK-BY-MARK

2-3 mark question: 'Rationalise the denominator of $1/(a + b\sqrt{x})$ and simplify.'

1 WRITE AND MULTIPLY BY THE CONJUGATE (1 m)

Identify the conjugate of the denominator: for $(3 + \sqrt{2})$ it is $(3 - \sqrt{2})$. Multiply BOTH numerator and denominator by this conjugate: $1/(3+\sqrt{2}) \times (3-\sqrt{2})/(3-\sqrt{2})$. State explicitly 'multiplying numerator and denominator by the conjugate $(3 - \sqrt{2})$'.

2 APPLY $(A+B)(A-B) = A^2 - B^2$ TO THE DENOMINATOR (1 m)

Denominator = $(3 + \sqrt{2})(3 - \sqrt{2}) = 3^2 - (\sqrt{2})^2 = 9 - 2 = 7$. Show the identity being used — this is where the surd is removed and the marks live.

3 WRITE THE SIMPLIFIED RATIONAL-DENOMINATOR RESULT (1 m)

Result = $(3 - \sqrt{2})/7$. Leave the answer with an integer (rational) denominator. If asked, state whether the result is rational or irrational: $(3 - \sqrt{2})/7$ is irrational because $\sqrt{2}$ is irrational.

TOPPER TEMPLATE · MARK-BY-MARK

2-3 mark question: 'Express the recurring decimal 0.bar(d) (e.g. 0.bar(3) or 1.2bar(7)) in

1 — LET X = THE DECIMAL; MULTIPLY TO SHIFT THE REPEATING BLOCK (1 m)

Let x = 0.3333... Since one digit repeats, multiply by 10: 10x = 3.3333... Clearly state how many places you shift (one digit → ×10, two digits → ×100).

2 — SUBTRACT TO ELIMINATE THE RECURRING PART (1 m)

Subtract the original from the shifted: 10x − x = 3.3333... − 0.3333... ⇒ 9x = 3. The repeating tails cancel exactly — this is the key idea examiners look for.

3 — SOLVE FOR X AND REDUCE TO LOWEST TERMS (1 m)

9x = 3 ⇒ x = 3/9 = 1/3. Always reduce p/q to its lowest form. State the conclusion: 'Hence 0.bar(3) = 1/3, which is rational.'

PYQ PATTERNS

Top PYQ patterns to drill

#1	Rationalise the denominator of $1/(a + b\sqrt{x})$ and simplify. (2-3 marks)	Almost every annual paper + SQP 2022, 2023, 2024
#2	Express the recurring decimal 0.bar(3) or 1.27bar in the form p/q. (2-3 marks)	SQP 2023, Annual Pattern 2022, 2024
#3	Without actual division, state whether the given decimal expansion is terminating or non-terminating; or classify a number as rational/irrational with reason. (1-2 marks)	Annual Pattern 2021, 2023; SQP 2024
#4	Simplify using laws of exponents, e.g. (64)^(1/2)·(64)^(1/3) or 2^(2/3)·2^(1/5). (2-3 marks)	SQP 2022, 2023; Annual Pattern 2024
#5	Represent $\sqrt{x}$ (e.g. $\sqrt{9.3}$ or $\sqrt{5}$) on the number line / Find a rational and an irrational number between two given numbers. (2-3 marks)	Annual Pattern 2021, 2022, 2023

RECAP · MEMORISE THESE

5-line revision

1 Two camps — Rational = p/q with $q \neq 0$ (terminating OR recurring decimal). Irrational = non-terminating, non-recurring. Together = real numbers, filling the line with no gaps.

2 Locate & classify — Use Pythagoras + compass to place $\sqrt{x}$; use the decimal-expansion test (does it repeat?) to classify any number.

3 Two key skills — Rationalise by the CONJUGATE ($a^2 - b^2$ clears the surd); apply exponent laws — ADD powers when multiplying, MULTIPLY powers when raising a power to a power.

WHAT'S NEXT

What's next

→

- → Chapter 2 — Polynomials (uses the surd-arithmetic and identities you practised here).
- → Sit the 15-MCQ Quick Drill (companion PDF) — under 20 minutes, target ⩾ 12/15.
- → Then the full Annual-Pattern Paper — 30 marks, internal-exam style, with full model answers.

You've stretched 'number' to fill the whole line.

Now prove it. Take the drill, sit the paper, beat the chapter.

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