CHAPTER 2

Polynomials

CBSE · Mathematics · Class 9

WHAT THIS CHAPTER DOES

Boards prep that builds confidence, not anxiety.

TODAY'S MISSION

Today's Mission

WHY THIS MATTERS

Where Polynomials Sit in Your Maths Journey

TOPIC

A

Part A — What is a Polynomial?

TOPIC

Polynomial in One Variable

POINT 1	POINT 2	POINT 3	POINT 4

TOPIC

Is It a Polynomial? Quick Filter

TOPIC

Classification by Number of Terms

POINT 1	POINT 2	POINT 3

TOPIC

Classification by Degree

TRY IT · SOLVE BEFORE YOU PEEK

30-Second Check

Work it out before you flip the answer.

SOLUTION

TOPIC

B

Part B — Zeroes, Remainder and Factor Theorems

THEOREM · LOAD-BEARING RESULT

Zero of a Polynomial

> A real number c is called a zero of polynomial p(x) if p(c) = 0.

STATEMENT

A real number c is called a zero of polynomial p(x) if p(c) = 0.

WHY THIS MATTERS

- Zeroes are where the polynomial vanishes
- They are also where the graph of y = p(x) crosses the x-axis.

WATCH OUT FOR

NOTE A linear polynomial has exactly ONE zero. A quadratic has at most 2. A cubic has at most 3.

WORKED EXAMPLE

Find the Zero of 3x + 2

1. Set p(x) = 0, so 3x + 2 = 0
2. Solve for x: 3x = -2
3. x = -2/3
4. So the zero of 3x + 2 is -2/3.

THEOREM · LOAD-BEARING RESULT

Remainder Theorem

> If a polynomial p(x) of degree at least 1 is divided by the linear polynomial (x - a), then the remainder is p(a).

STATEMENT

If a polynomial p(x) of degree at least 1 is divided by the linear polynomial (x - a), then the remainder is p(a).

WHY THIS MATTERS

- It saves the entire long-division process
- Instead of dividing, you just substitute a into p(x).

WATCH OUT FOR

NOTE Works ONLY for linear divisors of the form (x - a). For (x + a), substitute x = -a in p(x).

WORKED EXAMPLE

Remainder when p(x) = x^3 + 3x^2 - 5x + 7 is divided by (x - 2)

1. By Remainder Theorem, remainder = p(2)
2. p(2) = (2)^3 + 3(2)^2 - 5(2) + 7
3. = 8 + 12 - 10 + 7
4. = 17
5. Therefore remainder = 17.

THEOREM · LOAD-BEARING RESULT

Factor Theorem

> (x - a) is a factor of polynomial p(x) if and only if p(a) = 0.

STATEMENT

(x - a) is a factor of polynomial p(x) if and only if p(a) = 0.

WHY THIS MATTERS

- It converts the question 'is (x - a) a factor?' into a one-second substitution
- Combined with rational-root guessing, it cracks any cubic that has at least one integer root.

WATCH OUT FOR

NOTE Factor Theorem says (x - a) is the factor — the LINEAR polynomial — not the number a itself.

WORKED EXAMPLE

Factorise x^3 - 6x^2 + 11x - 6 using Factor Theorem

1. Constant term = -6, so possible rational roots are plus or minus 1, 2, 3, 6
2. Test p(1) = 1 - 6 + 11 - 6 = 0, so (x - 1) is a factor
3. Divide: x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6)
4. Split: x^2 - 5x + 6 = (x - 2)(x - 3)
5. Final: x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)

TOPIC

C

Part C — Algebraic Identities

TOPIC

The Square Identities

POINT 1	POINT 2	POINT 3	POINT 4

TOPIC

The Cube Identities

THEOREM · LOAD-BEARING RESULT

The Master Identity

> " $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

STATEMENT

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

WHY THIS MATTERS

- Whenever the sum $x + y + z = 0$, the right factor on the right side becomes 0, which gives the elegant special case: $x^3 + y^3 + z^3 = 3xyz$.

WATCH OUT FOR

NOTE Memorise the signs: PLUS the three squares, MINUS the three cross-products.

WORKED EXAMPLE

Evaluate 95^2 using identity

1. Rewrite 95 = 100 - 5
2. (100 - 5)^2 using (a - b)^2 = a^2 - 2ab + b^2
3. = 100^2 - 2(100)(5) + 5^2
4. = 10000 - 1000 + 25
5. = 9025

TOPPER TEMPLATE · MARK-BY-MARK

Model 3-Mark Answer — Apply x^3+y^3+z^3-3xyz

1

2

3

4

PYQ PATTERNS

Top 5 PYQ Patterns — How Examiners Test Polynomials

MARKS DISTRIBUTION

Where the Marks Hide — Sub-topic Weightage

TOPIC

Pre-Exam Self-Check

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** (x + y)^2 = x^2 + y^2

✓ **CORRECT** (x + y)^2 = x^2 + 2xy + y^2 — the 2xy middle term is the most-forgotten piece in CBSE markers' reports.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** Degree of 5 is 0

✓ **CORRECT** Degree of any non-zero constant polynomial is 0; the zero polynomial has UNDEFINED degree.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** If p(a) = 0, then a is a factor of p(x)

✓ **CORRECT** If p(a) = 0, then (x - a) is a factor of p(x). The factor is the LINEAR expression, not the number itself.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** Remainder Theorem works for any divisor

✓ **CORRECT** Remainder Theorem applies ONLY when dividing by a linear polynomial of the form (x - a) or (ax - b).

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** $x^3 + y^3 = (x + y)^3$

✓ **CORRECT** $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$. The cube of a sum has 4 terms: $x^3 + 3x^2 y + 3xy^2 + y^3$.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** A trinomial always has degree 2

✓ **CORRECT** A trinomial has 3 TERMS — its degree can be anything. e.g. x^4 + x + 1 is a trinomial of degree 4.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** To split the middle term in ax^2 + bx + c, just split b into any two numbers that add to b

✓ **CORRECT** Split b into two numbers that ADD to b AND MULTIPLY to a*c.

TOPIC

Trap

TRAP → TRUTH

× **MISTAKE** p(x) = 1/x is a polynomial

✓ **CORRECT** A polynomial has only non-negative integer powers of x. 1/x = x^(-1) is NOT a polynomial.

PYQ PATTERNS

Top PYQ patterns to drill

#1	Factorise a cubic using Factor Theorem then split middle term (3 marks)	90%
#2	Apply x^3+y^3+z^3-3xyz identity, especially when x+y+z=0 (3 marks)	80%
#3	Remainder Theorem direct substitution (2 marks)	60%
#4	Evaluate (101)^3, (95)^2 etc. using identities (2 marks)	50%
#5	Split middle term of a quadratic (2 marks)	50%

RECAP · MEMORISE THESE

Summary — Polynomials in One Slide

1 Polynomial = non-negative integer exponents only

2 Classify by terms: mono/bi/trinomial; by degree: 0/1/2/3

3 Zero of p(x) = value c with p(c) = 0

4 Remainder Theorem: p(x) divided by (x-a) gives remainder p(a)

5 Factor Theorem: (x-a) factor if and only if p(a) = 0

6 9 identities are non-negotiable — drill till automatic

WHAT'S NEXT

Coming Up Next

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